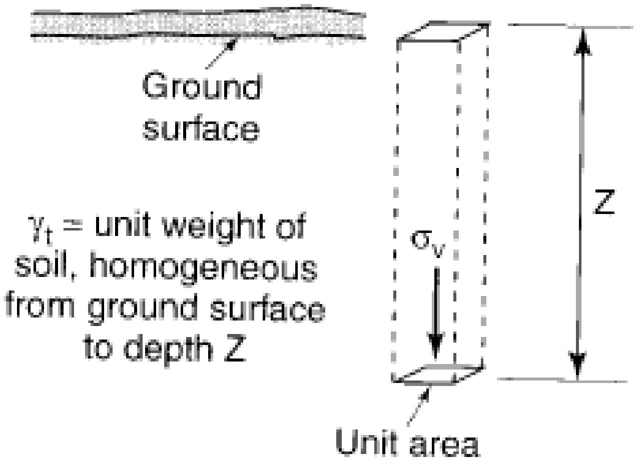
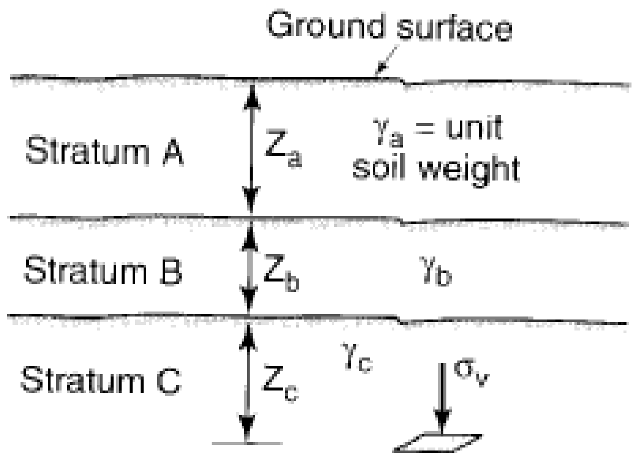
2.2 Ứng suất do trọng lượng bản thân

2.2.1 Ứng suất theo phương đứng

$$\sigma_v = \gamma_t Z$$

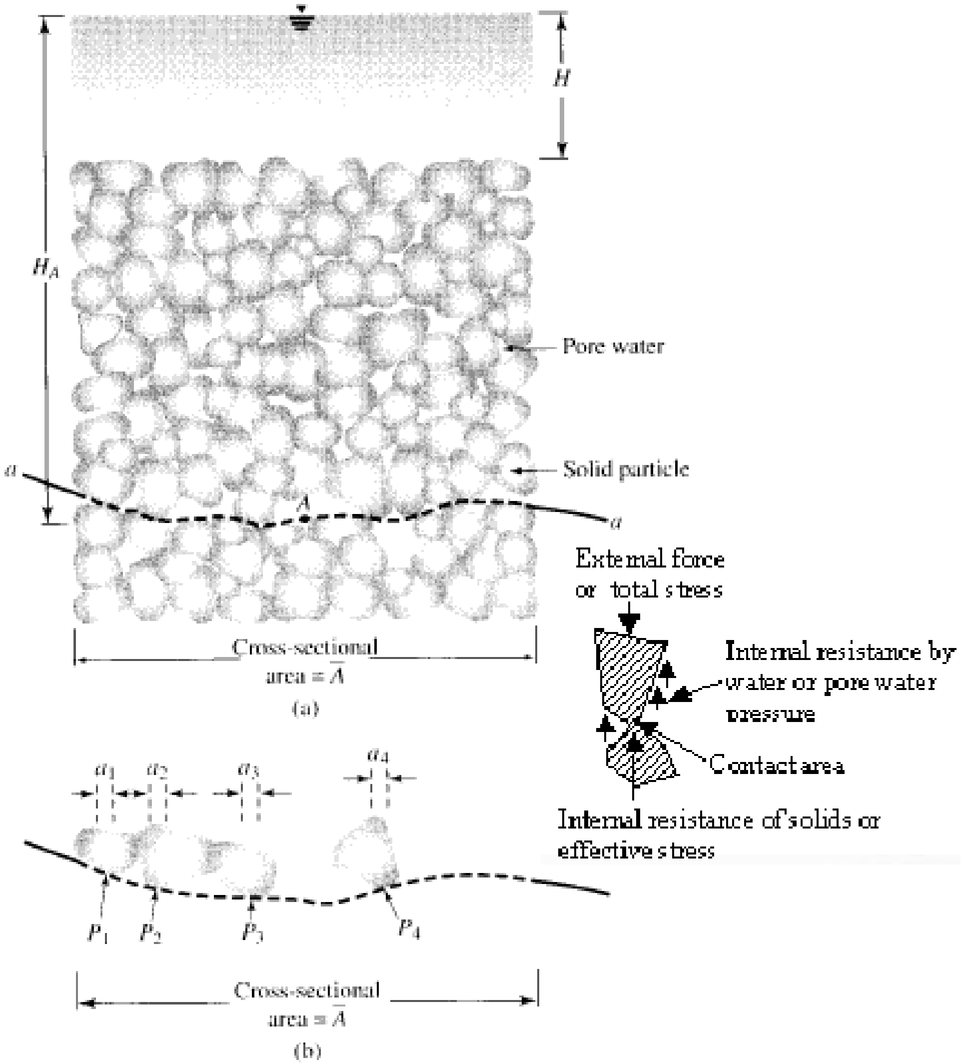


$\sigma_v = \gamma_t Z$



$\sigma_v = \gamma_a Z_a + \gamma_b Z_b + \gamma_c Z_c$

2.2.2 Ứng suất của phân tố đất bảo hòa



$$\sigma' = \frac{P_{1(v)} + P_{2(v)} + P_{3(v)} + \cdots + P_{n(v)}}{\bar{A}}$$

$$\sigma = \sigma' + \frac{u(\bar{A} - a_s)}{\bar{A}} = \sigma' + u(1 - a'_s)$$

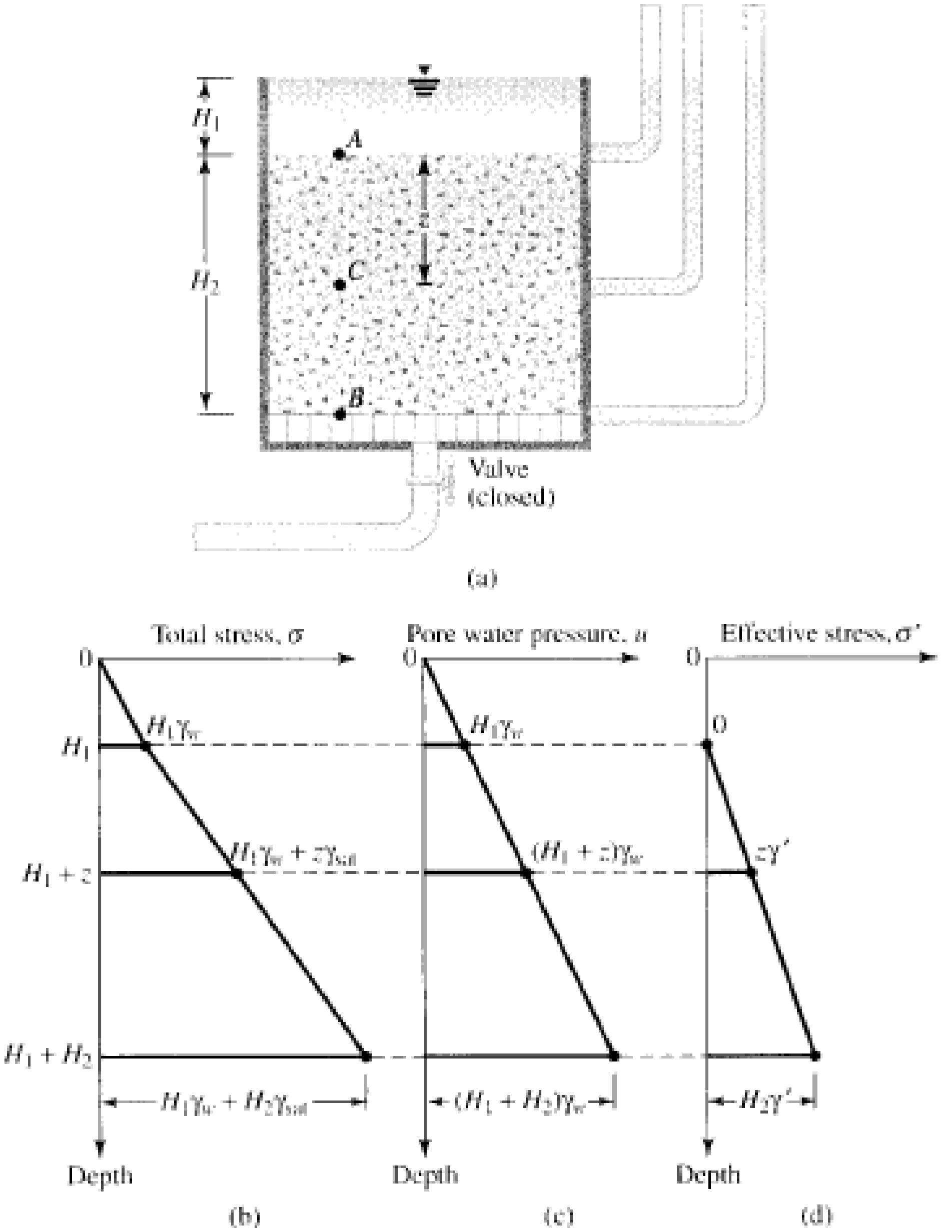
$a'_s \rightarrow 0$

$$a_s = a_1 + a_2 + a_3 + \cdots + a_n$$

$$\boxed{\sigma = \sigma' + u}$$

$$\begin{aligned} \sigma' &= [H\gamma_w + (H_A - H)\gamma_{sat}] - H_A\gamma_w \\ &= (H_A - H)(\gamma_{sat} - \gamma_w) \\ &= (\text{Height of the soil column}) \times \gamma' \end{aligned}$$

2 Ứng suất



H1
A
H2
z
C
B
Valve
(closed)
(a)
Total stress, σ
Pore water pressure, u
Effective stress, σ'
0
H1γw
H1γw + zγsat
H1 + z
H1 + H2
H1γw + H2γsat
(H1 + z)γw
(H1 + H2)γw
zγ'
H2γ'
Depth
(b)
(c)
(d)

2 Ứng suất

- **Thí dụ**

A soil profile is shown in Figure 8.3. Calculate the total stress, pore water pressure, and effective stress at *A*, *B*, *C*, and *D*.

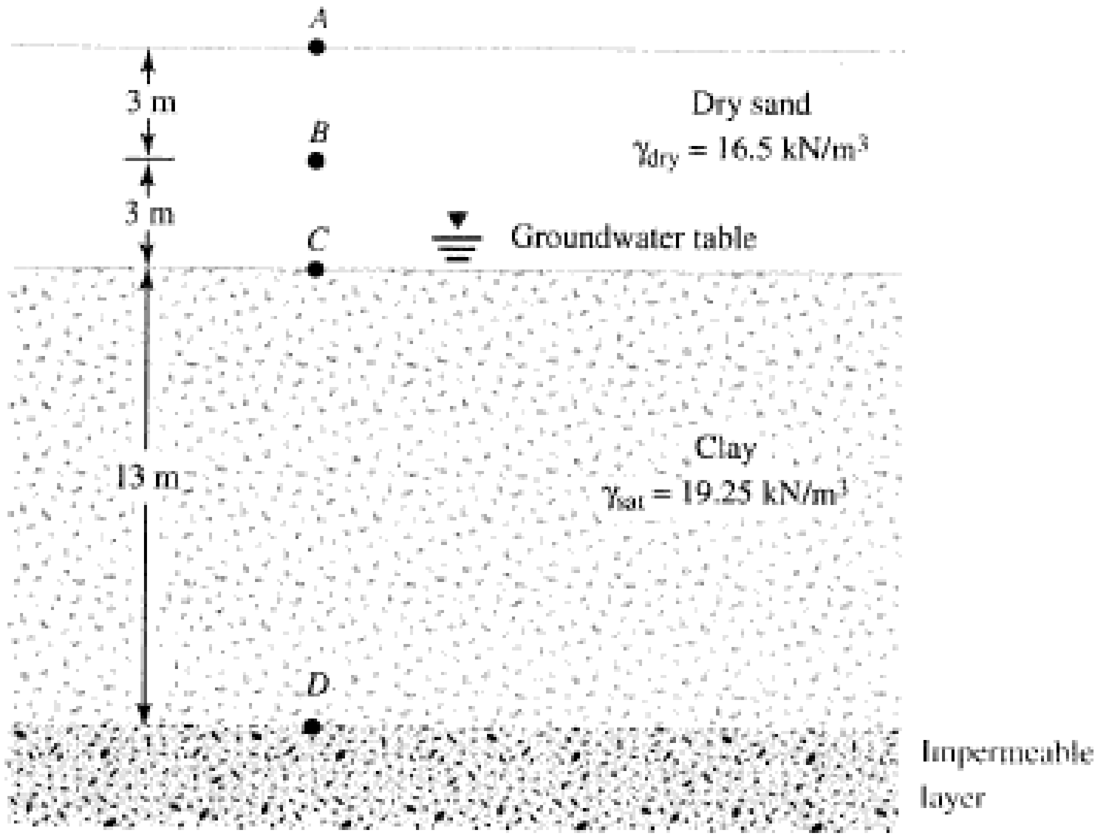
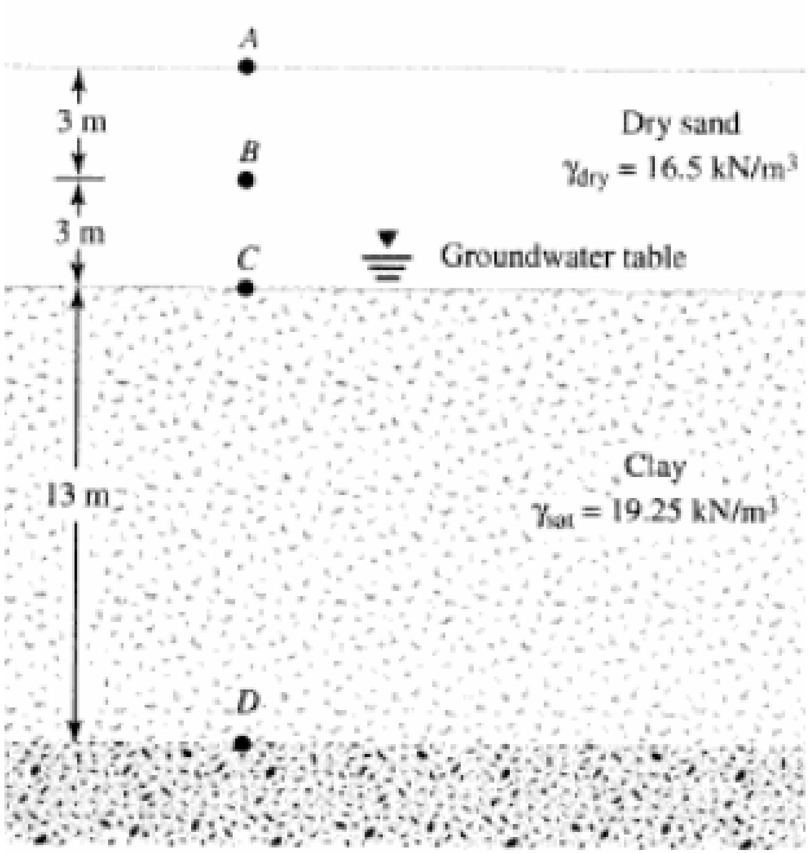


Figure 8.3 A soil profile for calculation of total stress, pore water pressure, and effective stress

2 Ứng suất

Solution



At *A*

total stress: $\sigma_A = 0$

pore water pressure: $u_A = 0$

effective stress: $\sigma'_A = 0$

At *B*

$$\sigma_B = 3\gamma_{dry(sand)} = 3 \times 16.5 = 49.5 \text{ kN/m}^2$$

$$u_B = 0 \text{ kN/m}^2$$

$$\sigma_B' = 49.5 - 0 = 49.5 \text{ kN/m}^2$$

At *C*

$$\sigma_C = 6\gamma_{dry(sand)} = 6 \times 16.5 = 99 \text{ kN/m}^2$$

$$u_C = 0 \text{ kN/m}^2$$

$$\sigma_C' = 99 - 0 = 99 \text{ kN/m}^2$$

At *D*

$$\sigma_D = 6\gamma_{dry(sand)} + 13\gamma_{sat(clay)}$$

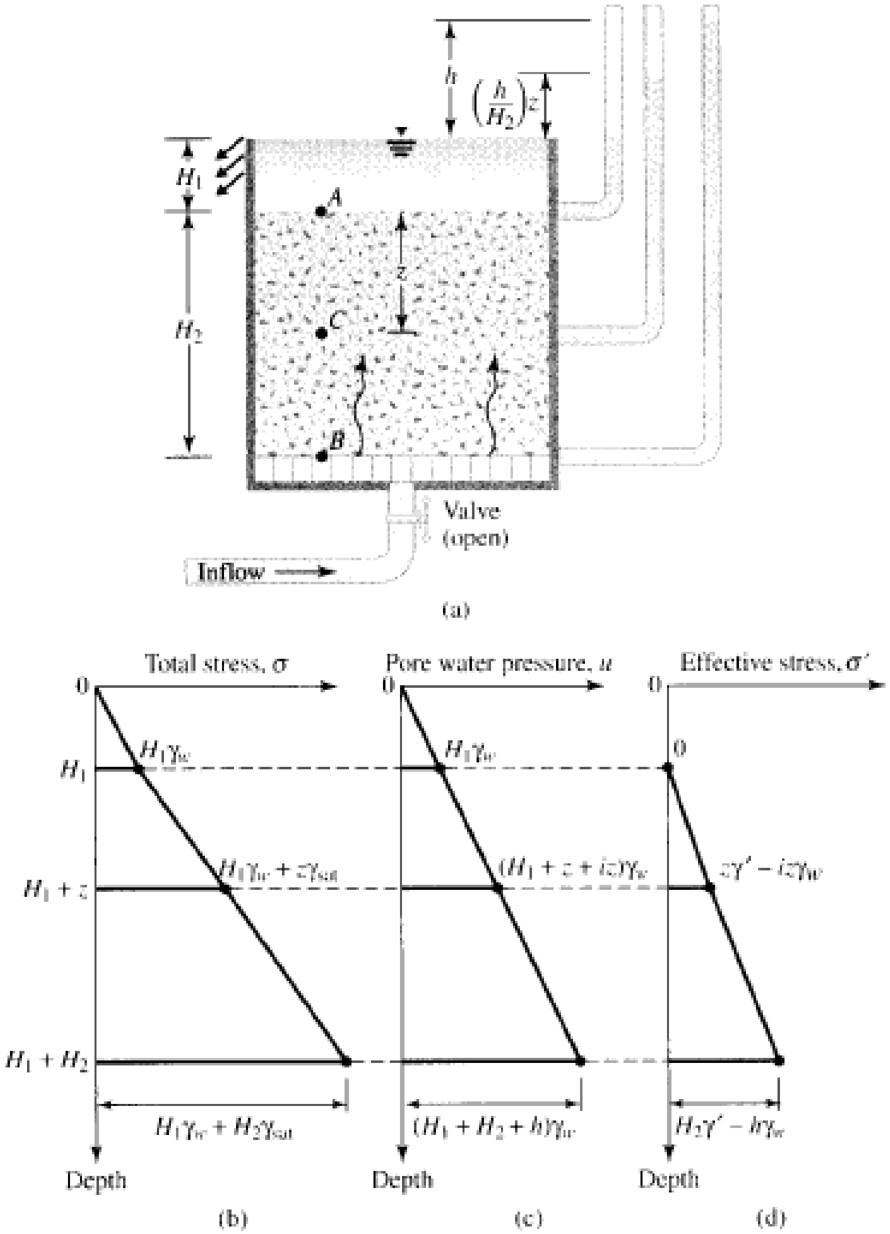
$$= 6 \times 16.5 + 13 \times 19.25$$

$$= 99 + 250.25 = 349.25 \text{ kN/m}^2$$

$$u_D = 13\gamma_w = 13 \times 9.81 = 127.53 \text{ kN/m}^2$$

$$\sigma_D' = 349.25 - 127.53 = 221.72 \text{ kN/m}^2$$

2 Ứng suất



$$\sigma'_C = \sigma_C - u_C$$
$$= z(\gamma_{sat} - \gamma_w) - \frac{h}{H_2} z\gamma_w$$
$$= z\gamma' - \frac{h}{H_2} z\gamma_w$$

$$\sigma'_C = z\gamma' - iz\gamma_w$$

$$\sigma'_c = z\gamma' - i_{cr} z\gamma_w = 0$$

$$i_{cr} = \frac{\gamma'}{\gamma_w}$$

Thí dụ

A 10-m thick layer of stiff saturated clay is underlain by a layer of sand (Figure 8.5). The sand is under artesian pressure. Calculate the maximum depth of cut H that can be made in the clay.

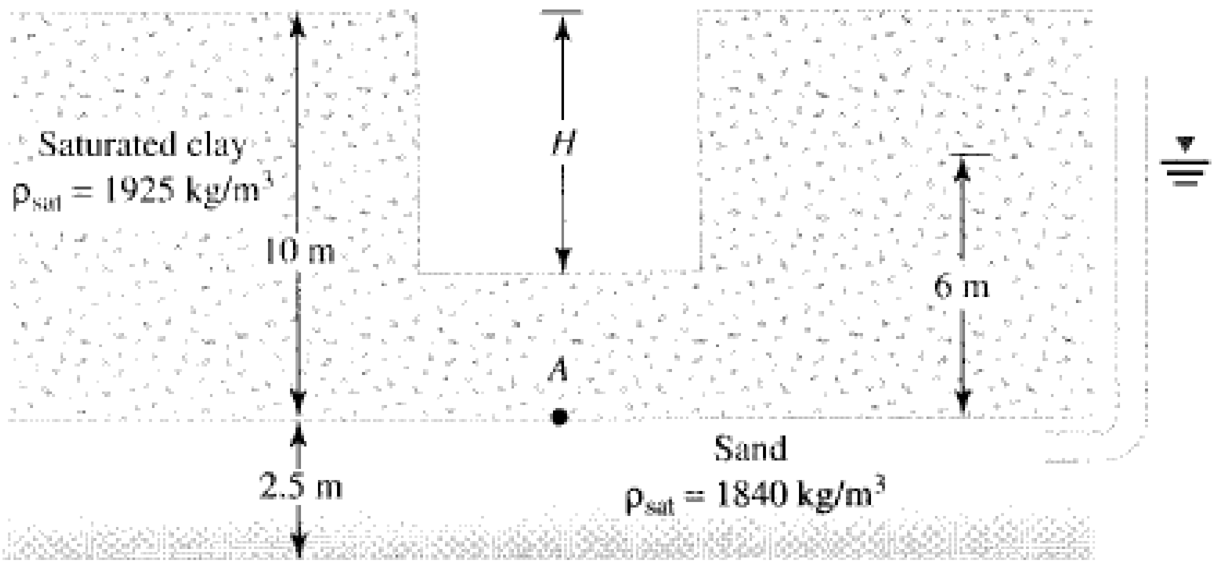
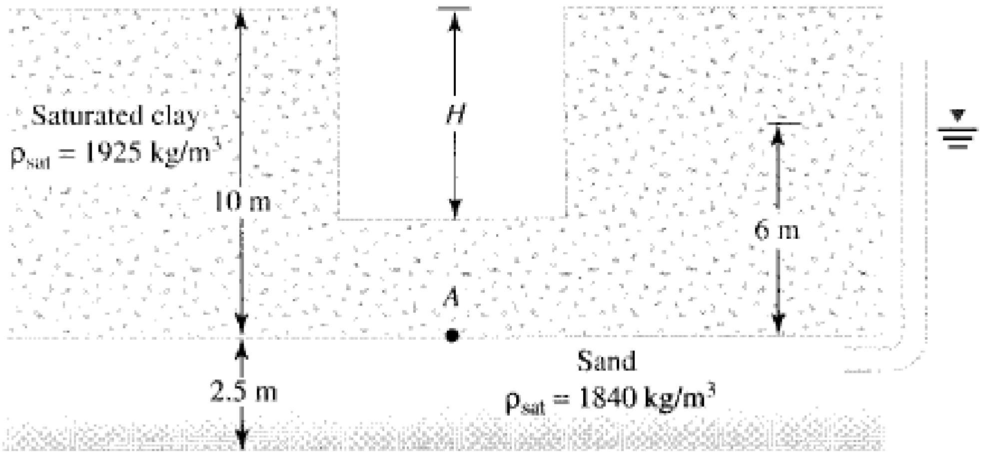


Figure 8.5

2 Ứng suất

Solution



$$\rho_{sat(clay)} = 1925\ \text{kg/m}^3,$$

$$\gamma_{sat(clay)} = \frac{1925 \times 9.81}{1000} = 18.88\ \text{kN/m}^3$$

$$\sigma_A = (10 - H)\gamma_{sat(clay)}$$

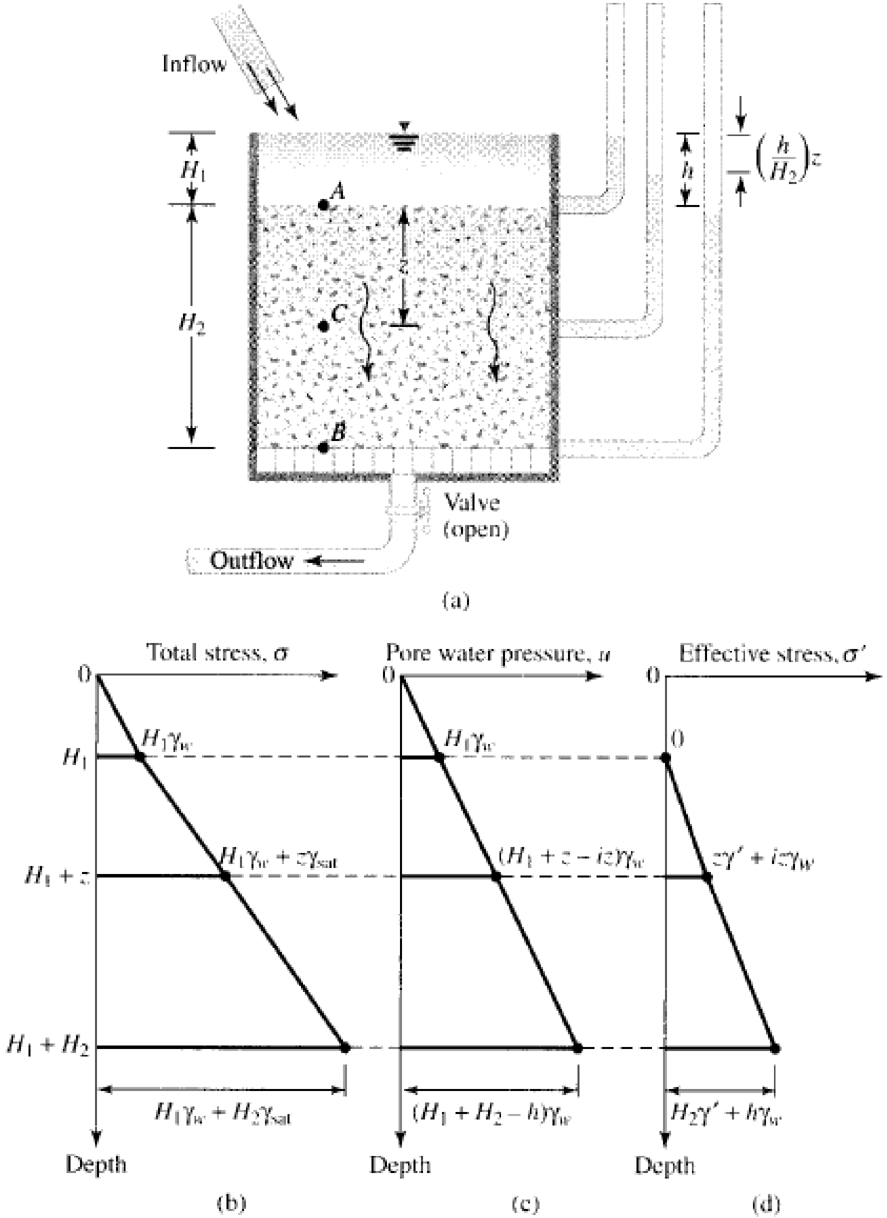
$$u_A = 6\gamma_w$$

$$\sigma_A - u_A = (10 - H)\gamma_{sat(clay)} - 6\gamma_w$$

$$(10 - H)18.88 - (6)9.81 = 0$$

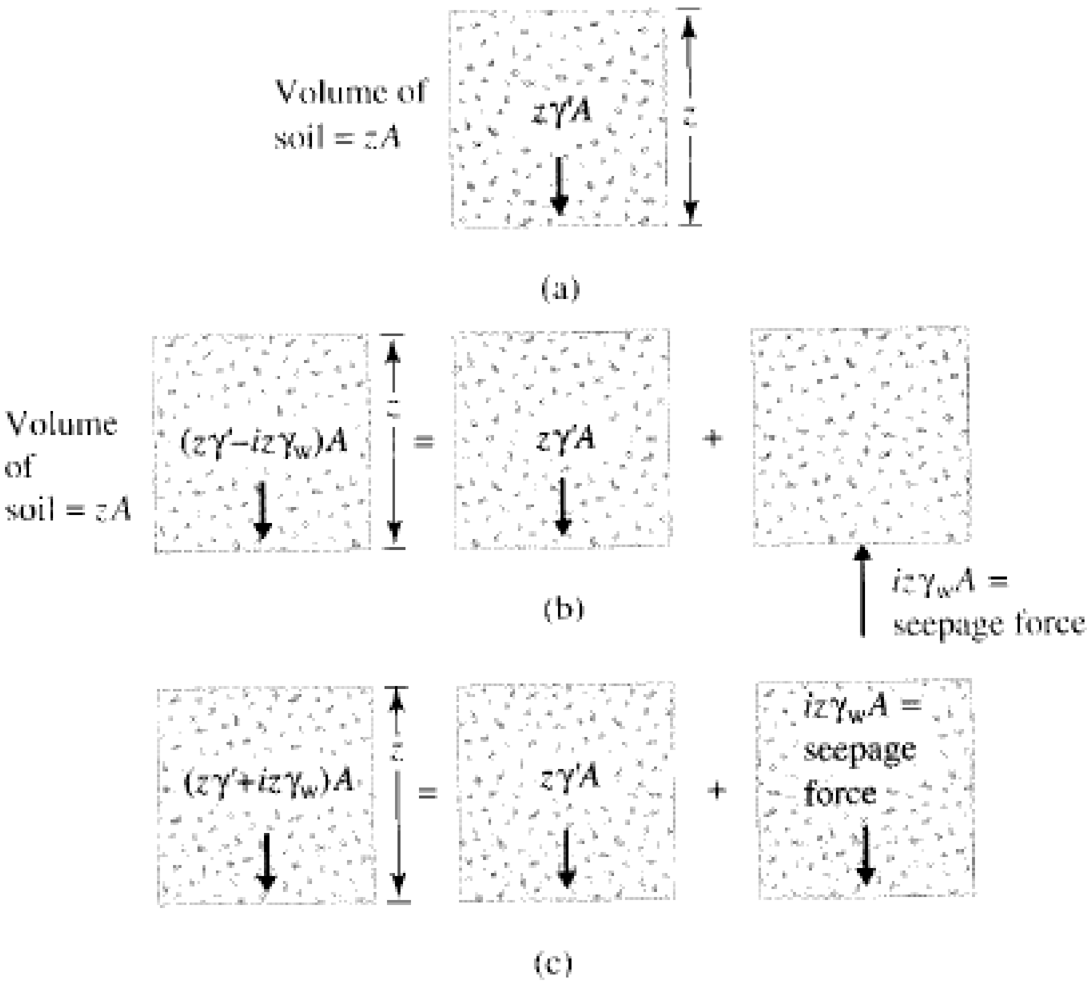
$$H = \frac{(10)18.88 - (6)9.81}{18.88} = \mathbf{6.88\ m}$$

2 Ứng suất

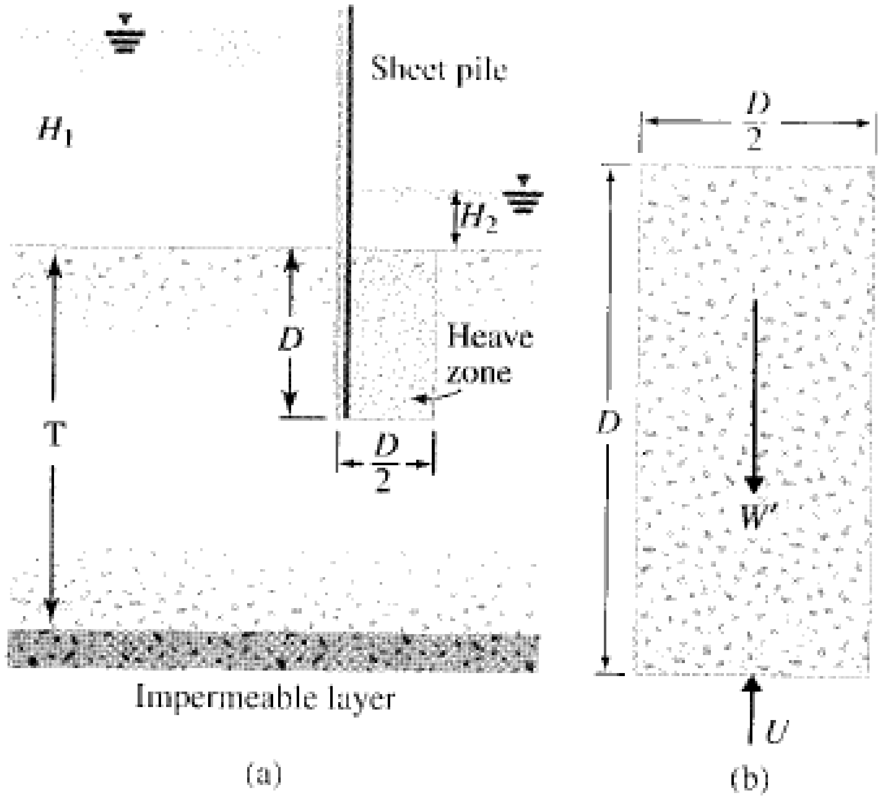

Inflow
H_1
H_2
A
C
B
z
h
$\left(\frac{h}{H_2}\right)z$
Valve
(open)
Outflow
(a)
Total stress, σ
Pore water pressure, u
Effective stress, σ'
0
H_1
$H_1 + z$
$H_1 + H_2$
$H_1\gamma_w$
$H_1\gamma_w + z\gamma_{sat}$
$H_1\gamma_w + H_2\gamma_{sat}$
$(H_1 + z - iz)\gamma_w$
$(H_1 + H_2 - h)\gamma_w$
$z\gamma' + iz\gamma_w$
$H_2\gamma' + h\gamma_w$
Depth
(b)
(c)
(d)

2 Ứng suất

- **Lực thấm**



2 Ứng suất



$$FS = \frac{\gamma'}{i_{av}\gamma_w}$$

2 Ứng suất

- **Thí dụ**

Consider the upward flow of water through a layer of sand in a tank as shown in Figure 8.9. For the sand, the following are given: void ratio (e) = 0.52 and specific gravity of solids = 2.67.

a. Calculate the total stress, pore water pressure, and effective stress at points A and B.
b. What is the upward seepage force per unit volume of soil?

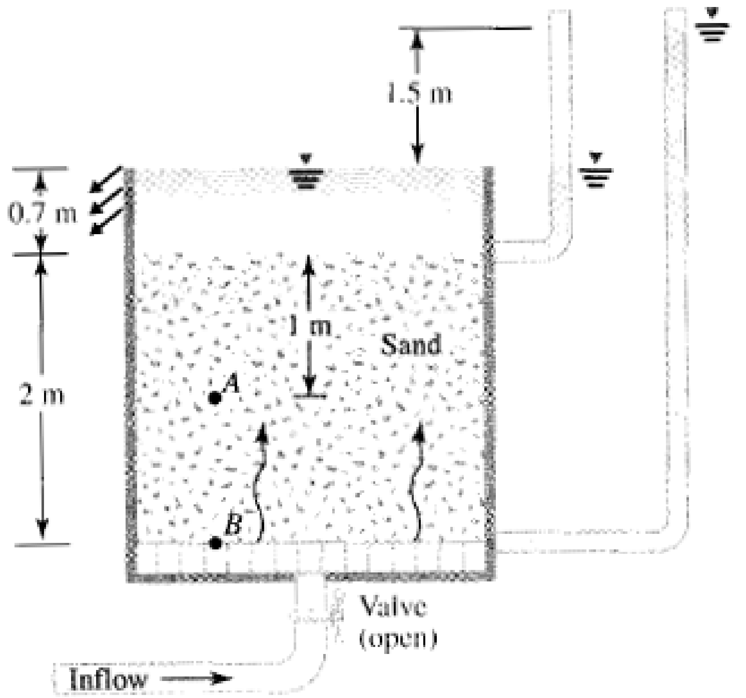
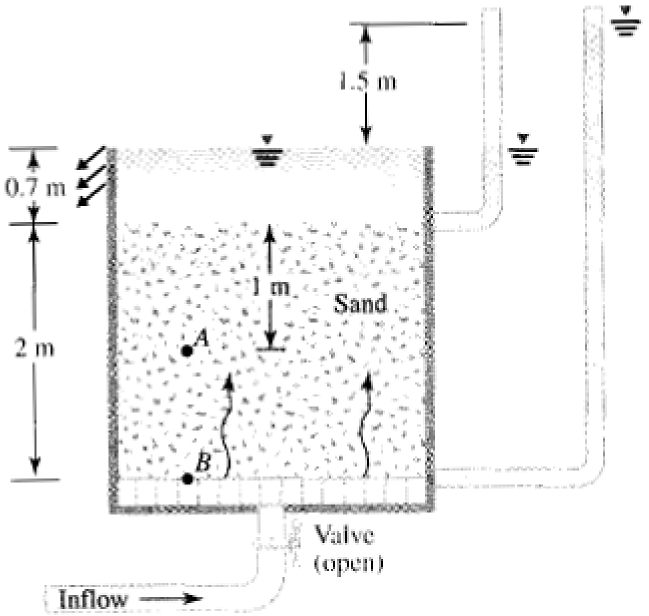


Figure 8.9 Upward flow of water through a layer of sand in a tank

Solution



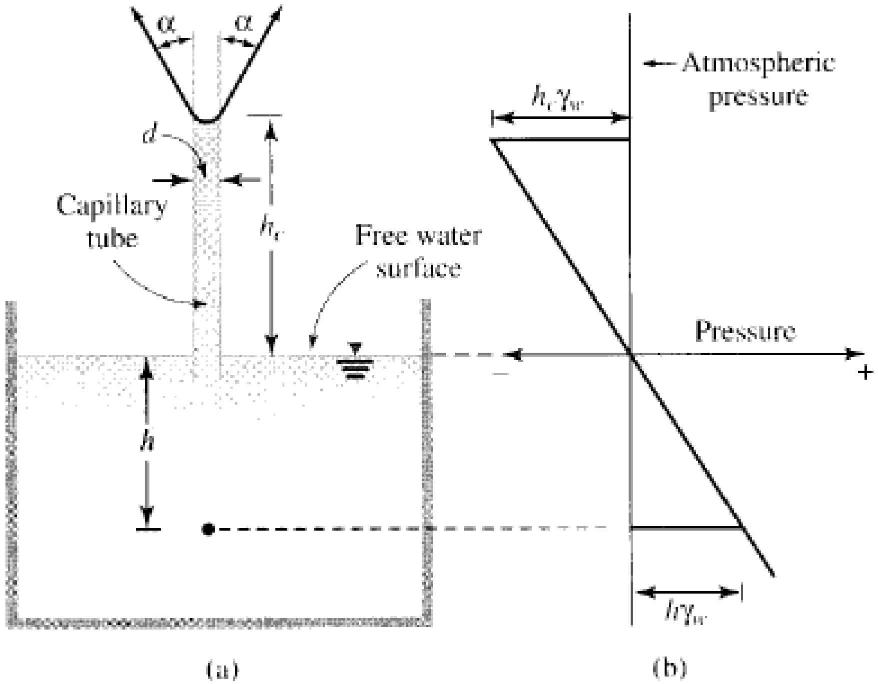
a. $$\gamma_{sat} = \frac{(G_s + e)\gamma_w}{1+e} = \frac{(2.67 + 0.52)9.81}{1 + 0.52} = 20.59 \text{ kN/m}^3$$

Point	Total stress, σ (kN/m²)	Pore water pressure, u (kN/m²)	Effective stress, $\sigma' = \sigma - u$ (kN/m²)
A	$0.7\gamma_w + 1\gamma_{sat} = (0.7)(9.81) + (1)(20.59) =$ **27.46**	$\left[(1+0.7) + \left(\frac{1.5}{2}\right)(1)\right]\gamma_w = (2.45)(9.81) =$ **24.03**	**3.43**
B	$0.7\gamma_w + 2\gamma_{sat} = (0.7)(9.81) + (2)(20.59) =$ **48.05**	$(2 + 0.7 + 1.5)\gamma_w = (4.2)(9.81) =$ **41.2**	**6.85**

b. Hydraulic gradient $(i) = 1.5/2 = 0.75$

$$i\gamma_w = (0.75)(9.81) = \mathbf{7.36\ kN/m^3}$$

Mao dẫn trong đất



$$\left(\frac{\pi}{4}d^2\right)h_c\gamma_w = \pi d T \cos\alpha$$

$$h_c = \frac{4T\cos\alpha}{d\gamma_w}$$

Ø **Đối với nước cất**

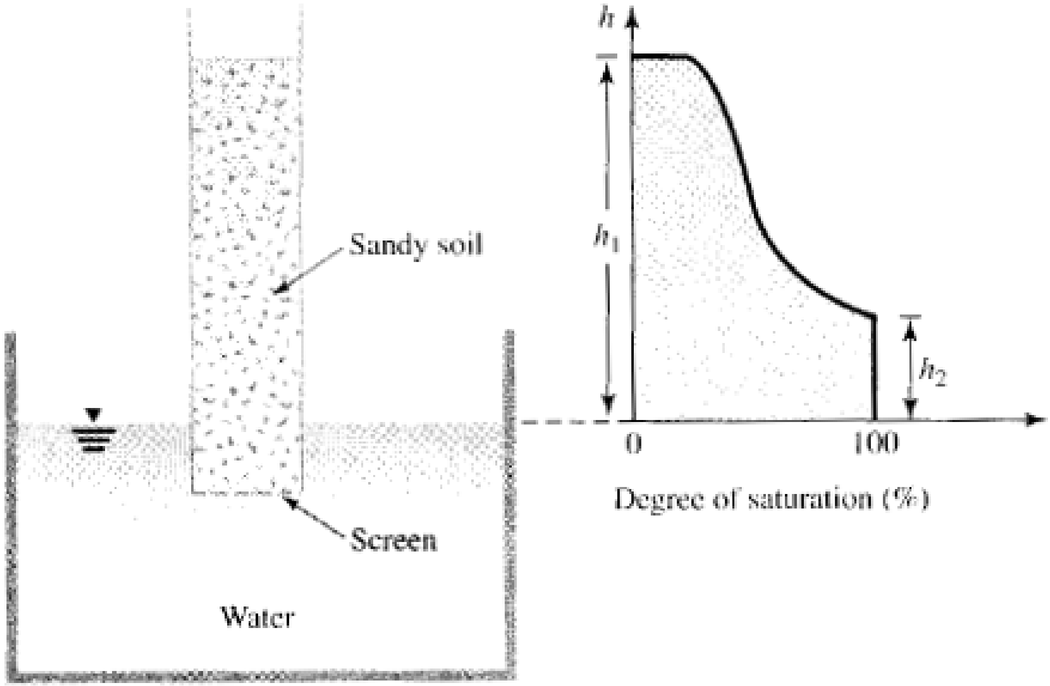
$$\alpha = 0.$$

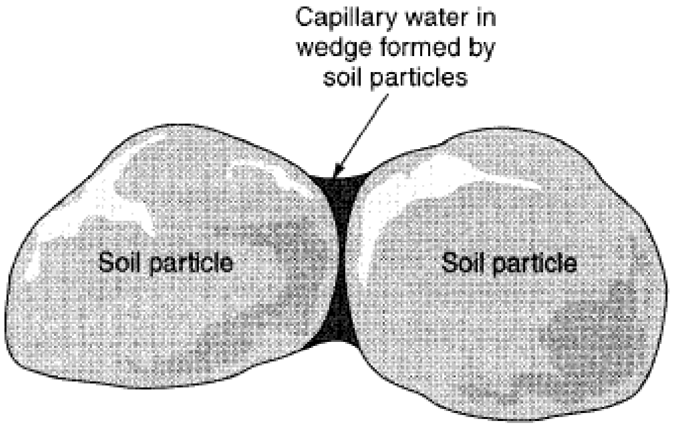
$$T = 72 \text{ mN/m.}$$

Soil type	Range of capillary rise	
	m	ft
Coarse sand	0.1–0.2	0.3–0.6
Fine sand	0.3–1.2	1–4
Silt	0.75–7.5	2.5–25
Clay	7.5–23	25–75

$$h_c = \frac{4T}{d\gamma_w} \Longrightarrow h_c \propto \frac{1}{d}$$

2 Ứng suất





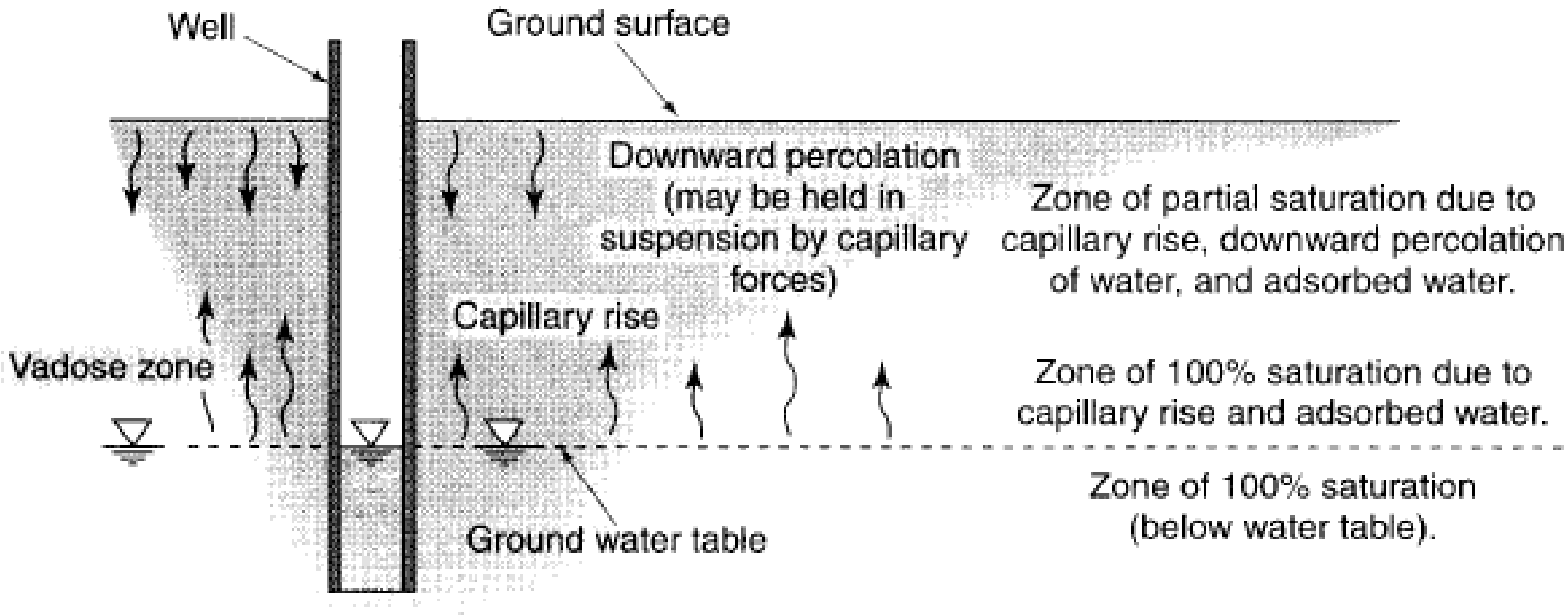
$$h_1\,(\text{mm}) = \frac{C}{eD_{10}}$$

where D_{10} = effective size (mm)

e = void ratio

C = a constant that varies from 10 to 50 mm^2

2 Ứng suất



Well
Ground surface
Downward percolation (may be held in suspension by capillary forces)
Zone of partial saturation due to capillary rise, downward percolation of water, and adsorbed water.
Capillary rise
Vadose zone
Zone of 100% saturation due to capillary rise and adsorbed water.
Zone of 100% saturation (below water table).
Ground water table

Thí dụ

A soil profile is shown in Figure 8.18. Note the zone of capillary rise in the sand layer overlying clay. In this zone, the average degree of saturation and the moist unit weight are 60% and 17.6 kN/m^3, respectively. Calculate and plot the variation of σ, u, and σ' with depth.

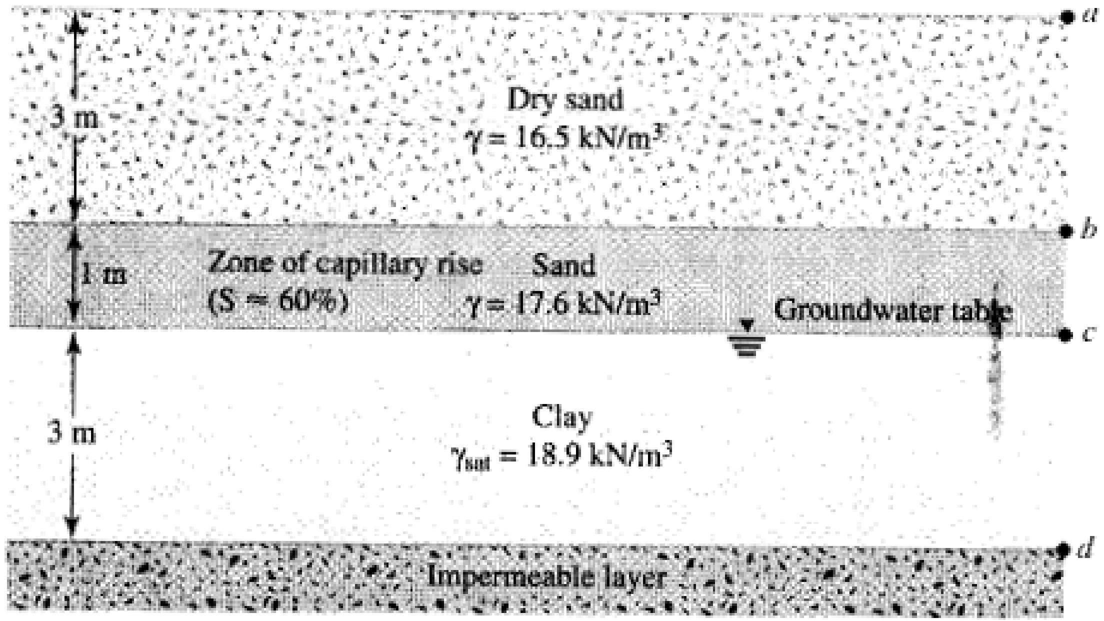
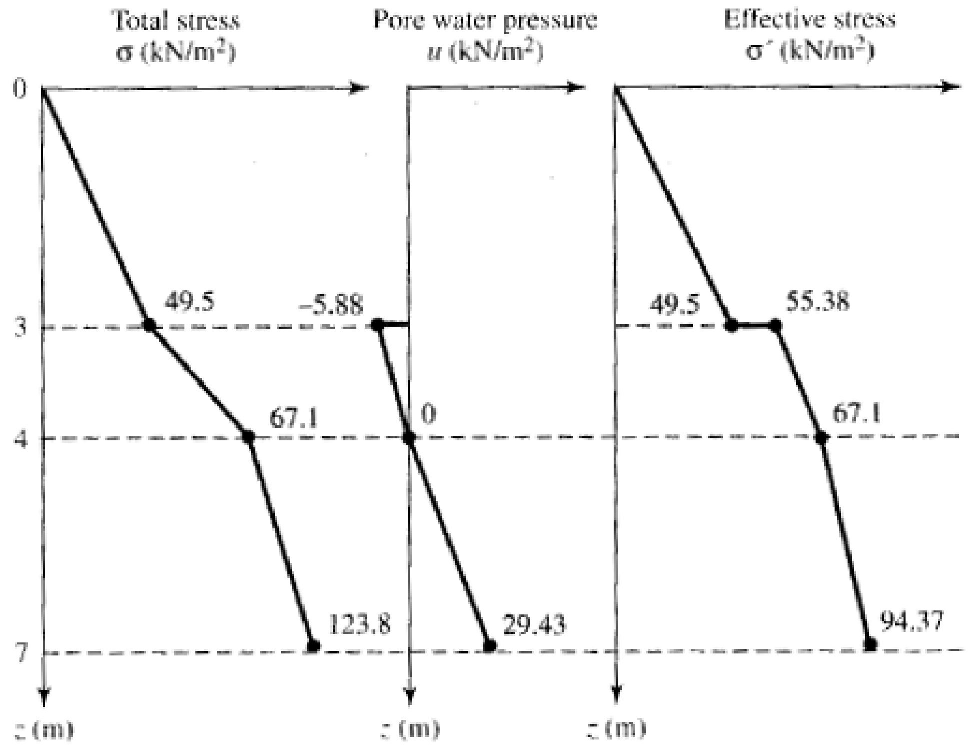


Figure 8.18 Soil profile with capillary rise

Solution

Depth below ground surface (m)	Total stress, σ (kN/m²)	Pore water pressure, u (kN/m²)	Effective stress, σ' (kN/m²)
0	0	0	0
3			
Immediately above the capillary zone	(3)(16.5) = 49.5	0	49.5
Just inside the capillary zone	(3)(16.5) = 49.5	$-(S\gamma_w)(1) = -(0.6)(9.81)(1) = -5.88$	55.38
4	(3)(16.5) + (1)(17.6) = 67.1	0	67.1
7	(3)(16.5) + (1)(17.6) + (3)(18.9) = 123.8	$3\gamma_w = (3)(9.81) = 29.43$	94.37



2 Ứng suất

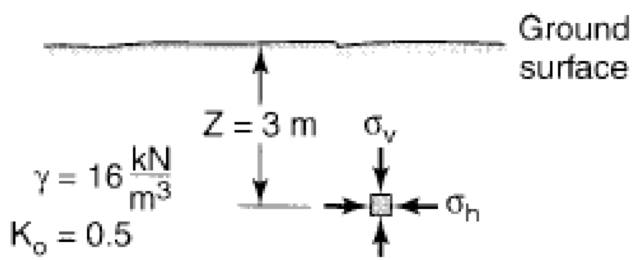
2.2.3 Ứng suất theo phương ngang

$$s_h = s'_h + u$$

$$s'_h = K s'_v$$

Ø K ở điều kiện tĩnh: K_o

Soil Type	K_0
Granular, loose	0.5–0.6
Granular, dense	0.3–0.5
Clay, soft	0.9–1.1 (undrained)
Clay, hard	0.8–0.9 (undrained)



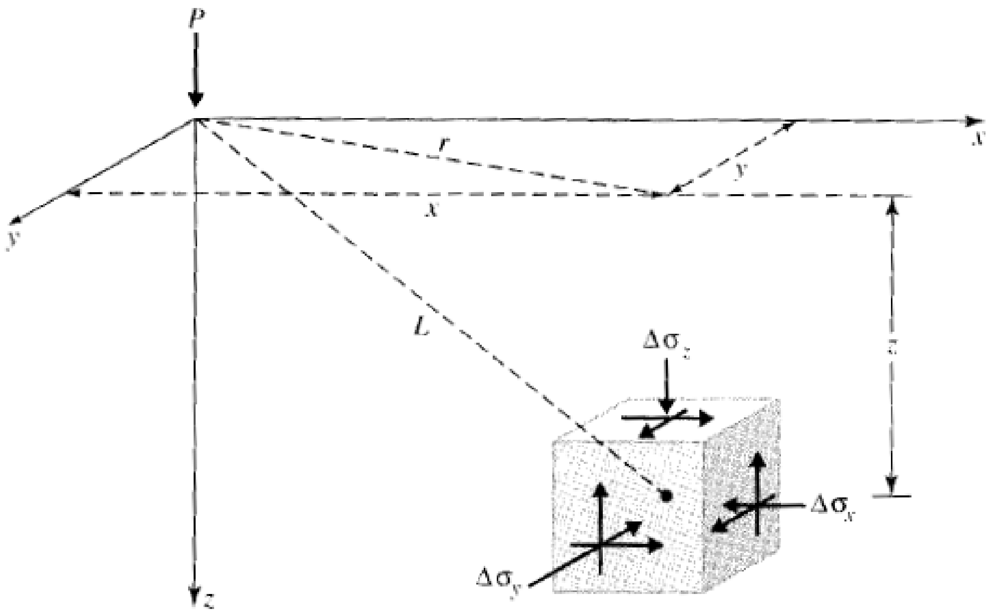
$$\sigma_v = \sigma Z = (16\ \text{kN/m}^3)(3\ m) = 48\ kN/m^2 = 48\ \text{kPa}$$

$$\sigma_h = K_0 \sigma_v = (0.5)(48\ \text{kPa}) = 24\ \text{kPa}$$

2 Ứng suất

2.3 Ứng suất do tải trọng ngòai

2.3.1 Ứng suất do tải trọng tập trung



$$\Delta\sigma_z = \frac{3P}{2\pi}\frac{z^3}{L^5} = \frac{3P}{2\pi}\frac{z^3}{(r^2+z^2)^{5/2}}$$

$$\Delta\sigma_x = \frac{P}{2\pi}\left\{\frac{3x^2z}{L^5} - (1-2\mu)\left[\frac{x^2-y^2}{Lr^2(L+z)} + \frac{y^2z}{L^3r^2}\right]\right\}$$

$$\Delta\sigma_y = \frac{P}{2\pi}\left\{\frac{2y^2z}{L^5} - (1-2\mu)\left[\frac{y^2-x^2}{Lr^2(L+z)} + \frac{x^2z}{L^3r^2}\right]\right\}$$

where $r = \sqrt{x^2+y^2}$

$L = \sqrt{x^2+y^2+z^2} = \sqrt{r^2+z^2}$

μ = Poisson's ratio

$$t_{zy} = -\frac{3.p}{2.p}\frac{y.z^2}{r^5} \qquad t_{zx} = -\frac{3.p}{2.p}\frac{x.z^2}{r^5}$$

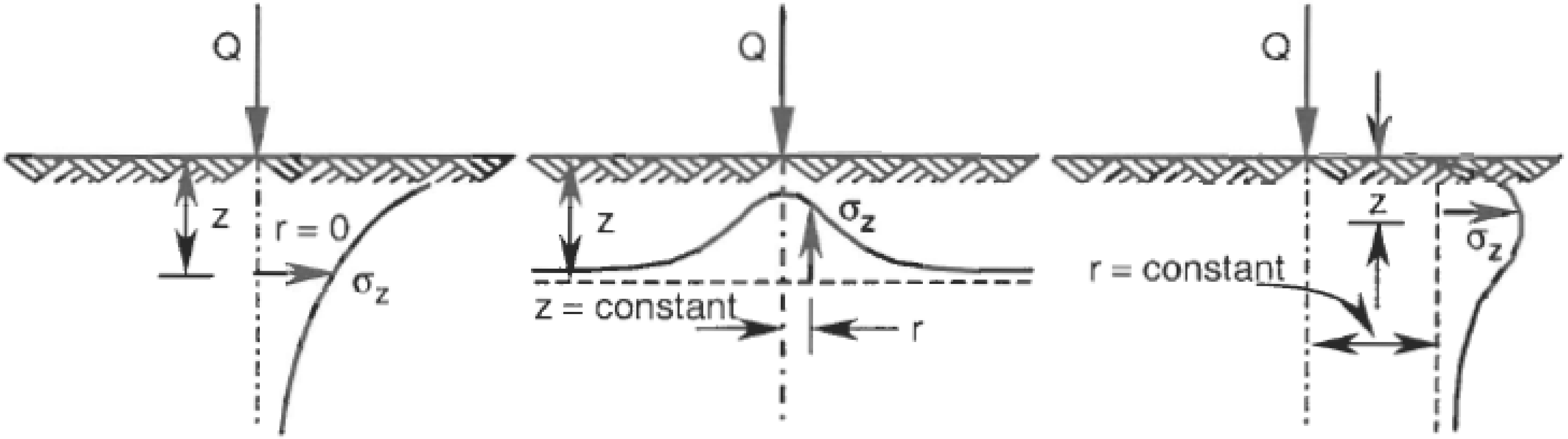
$$t_{xy} = \frac{3.p}{2.p}\left[\frac{x.y.z}{r^5} - \frac{1-2.m}{3}.\frac{x.y(2.r+z)}{r^3(R+z)^2}\right]$$

$$\Delta\sigma_z = \frac{P}{z^2}\left\{\frac{3}{2\pi}\frac{1}{[(r/z)^2+1]^{5/2}}\right\} = \frac{P}{z^2}I_1$$

$$I_1 = \frac{3}{2\pi}\frac{1}{[(r/z)^2+1]^{5/2}}$$

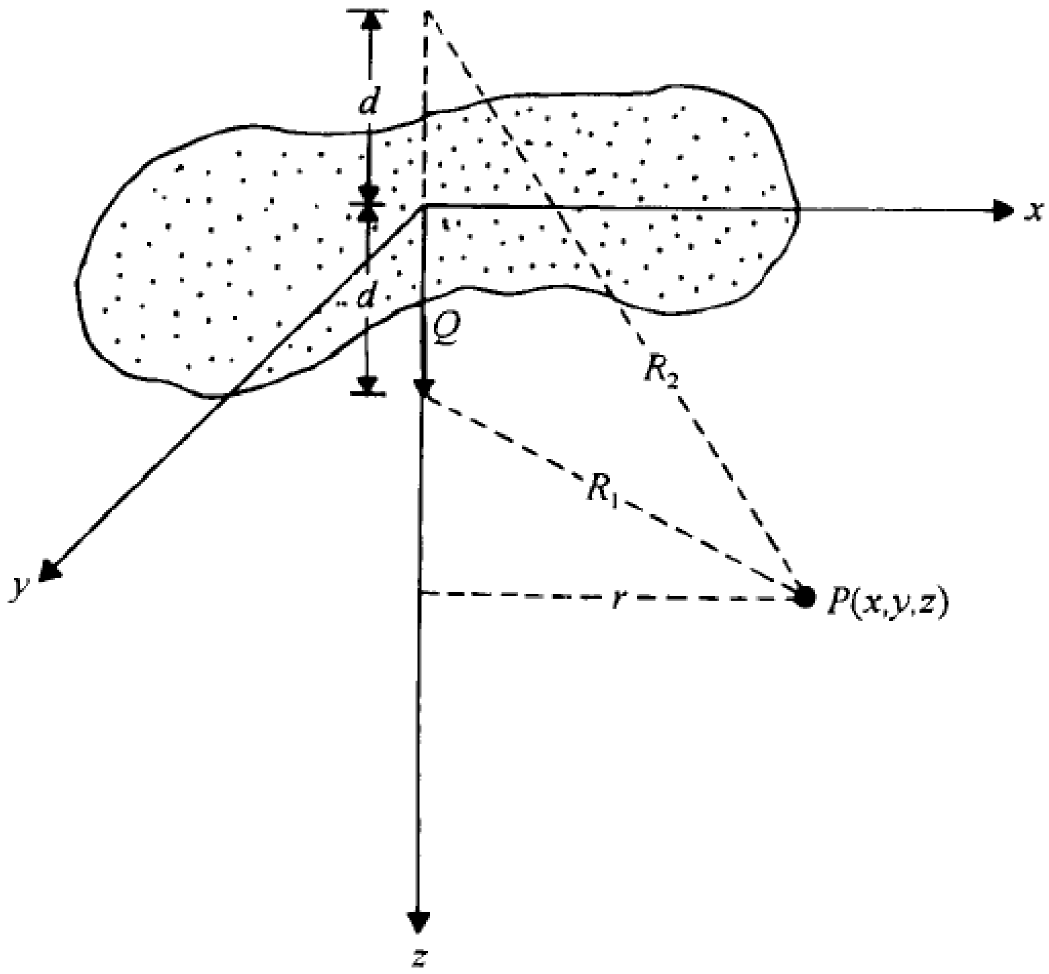
r/z	I_1	r/z	I_1
0	0.4775	0.75	0.1565
0.02	0.4770	0.80	0.1386
0.04	0.4765	0.85	0.1226
0.06	0.4723	0.90	0.1083
0.08	0.4699	0.95	0.0956
0.10	0.4657	1.00	0.0844
0.12	0.4607	1.20	0.0513
0.14	0.4548	1.40	0.0317
0.16	0.4482	1.60	0.0200
0.18	0.4409	1.80	0.0129
0.20	0.4329	2.00	0.0085
0.22	0.4242	2.20	0.0058
0.24	0.4151	2.40	0.0040
0.26	0.4050	2.60	0.0029
0.28	0.3954	2.80	0.0021
0.30	0.3849	3.00	0.0015
0.32	0.3742	3.20	0.0011
0.34	0.3632	3.40	0.00085
0.36	0.3521	3.60	0.00066
0.38	0.3408	3.80	0.00051
0.40	0.3294	4.00	0.00040
0.45	0.3011	4.20	0.00032
0.50	0.2733	4.40	0.00026
0.55	0.2466	4.60	0.00021
0.60	0.2214	4.80	0.00017
0.65	0.1978	5.00	0.00014
0.70	0.1762		

2 Ứng suất



2 Ứng suất

- **Lực tác dụng bên trong khối đất**

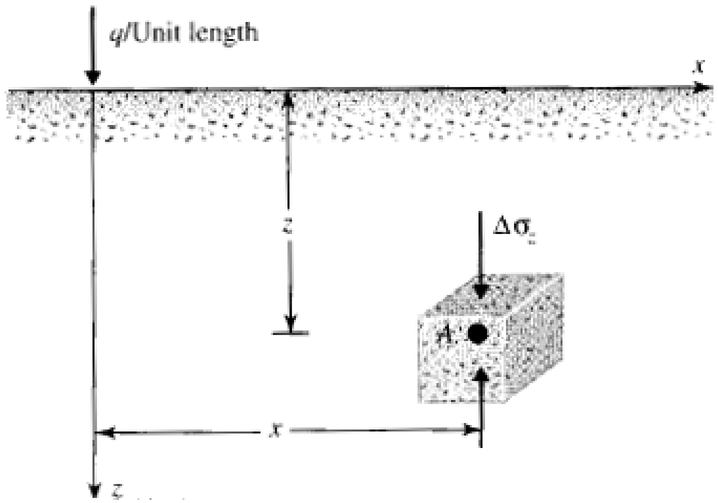


$$\sigma_z = \frac{Q}{2\pi \nu'^2 z^2 [1 + (r/\nu' z)^2]^{1.5}}$$

$$\nu' = \sqrt{\frac{1 - 2\nu}{2(1 - \nu)}}$$

2 Ứng suất

2.3.2 Ứng suất do tải trọng đường thẳng



x/z	$\Delta\sigma_z/(q/z)$	x/z	$\Delta\sigma_z/(q/z)$
0	0.637	1.3	0.088
0.1	0.624	1.4	0.073
0.2	0.589	1.5	0.060
0.3	0.536	1.6	0.050
0.4	0.473	1.7	0.042
0.5	0.407	1.8	0.035
0.6	0.344	1.9	0.030
0.7	0.287	2.0	0.025
0.8	0.237	2.2	0.019
0.9	0.194	2.4	0.014
1.0	0.159	2.6	0.011
1.1	0.130	2.8	0.008
1.2	0.107	3.0	0.006

$$\Delta s_z = \frac{2qz^3}{p\left(x^2+z^2\right)^2}$$

$$\Delta s_x = \frac{2qx^2z}{p\left(x^2+z^2\right)^2}$$

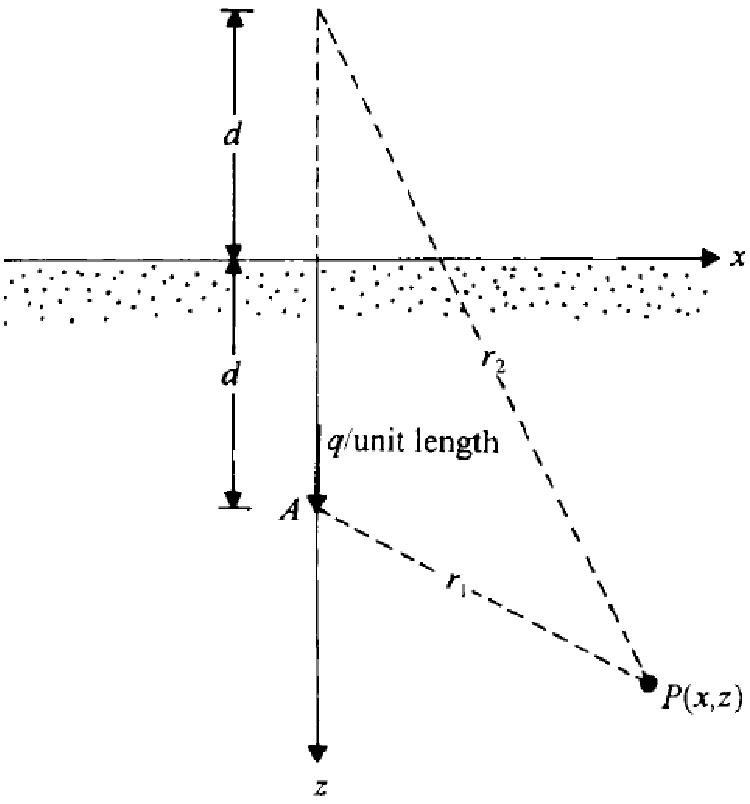
$$\Delta t_{zx} = \frac{2qz^2}{p\left(x^2+z^2\right)^2}$$

$$\Delta\sigma_z = \frac{2q}{\pi z[(x/z)^2+1]^2}$$

$$\frac{\Delta\sigma_z}{(q/z)} = \frac{2}{\pi[(x/z)^2+1]^2}$$

2 Ứng suất

Tải trọng đường thẳng bên trong khối đất

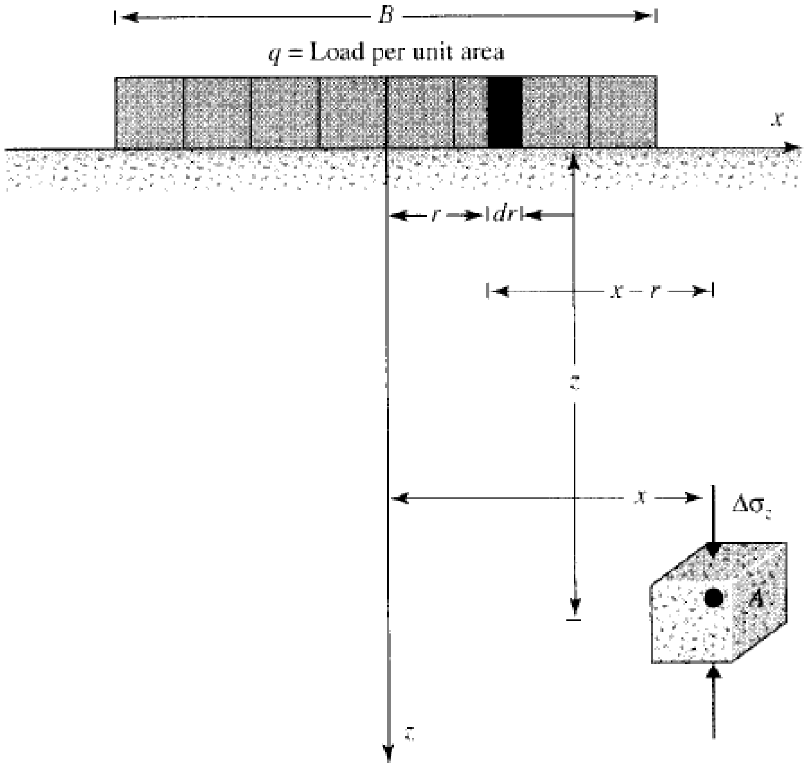


$$\sigma_z = \frac{q}{\pi}\left(\frac{1}{2(1-\nu)}\left\{\frac{(z-d)^3}{r_1^4} + \frac{(z+d)[(z+d)^2+2dz]}{r_2^4} - \frac{8dz(d+z)x^2}{r_2^6}\right\} + \frac{1-2\nu}{4(1-\nu)}\left(\frac{z-d}{r_1^2} + \frac{3z+d}{r_2^4} - \frac{4zx^2}{r_2^4}\right)\right)$$

$$\sigma_x = \frac{q}{\pi}\left\{\frac{1}{2(1-\nu)}\left[\frac{(z-d)x^2}{r_1^4} + \frac{(z+d)(x^2+2d^2)-2dx^2}{r_2^4} + \frac{8dz(d+z)x^2}{r_2^6}\right] + \frac{1-2\nu}{4(1-\nu)}\left(\frac{d-z}{r_1^2} + \frac{z+3d}{r_2^2} + \frac{4zx^2}{r_2^4}\right)\right\}$$

$$\tau_{xz} = \frac{qx}{\pi}\left\{\frac{1}{2(1-\nu)}\left[\frac{(z-d)^2}{r_1^4} + \frac{z^2-2dz-d^2}{r_2^4} + \frac{8dz(d+z)^2}{r_2^6}\right] + \frac{1-2\nu}{4(1-\nu)}\left[\frac{1}{r_1^2} - \frac{1}{r_2^2} + \frac{4z(d+z)}{r_2^4}\right]\right\}$$

2.3.3 Ứng suất do tải trọng phần bô đều hình băng

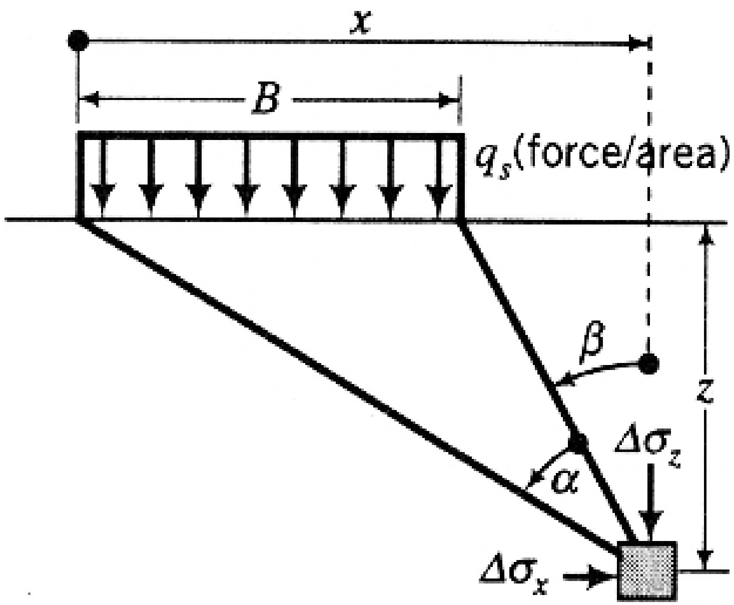


$$d\sigma_z = \frac{2(q\,dr)z^3}{\pi[(x-r)^2+z^2]^2}$$

$$\Delta\sigma_z = \int d\sigma_z = \int_{-B/2}^{+B/2}\left(\frac{2q}{\pi}\right)\left\{\frac{z^3}{[(x-r)^2+z^2]^2}\right\}dr$$

$$\Delta\sigma_z = \frac{q}{\pi}\left\{\tan^{-1}\left[\frac{z}{x-(B/2)}\right] - \tan^{-1}\left[\frac{z}{x+(B/2)}\right] - \frac{Bz[x^2-z^2-(B^2/4)]}{[x^2+z^2-(B^2/4)]^2+B^2z^2}\right\}$$

2 Ứng suất



$$\Delta\sigma_z = \frac{q}{\pi}\left[\alpha + \sin\alpha \;\cos(\alpha + 2\beta)\right]$$

$$\Delta\sigma_x = \frac{q}{\pi}\left[\alpha - \sin\alpha \;\cos(\alpha + 2\beta)\right]$$

$$\Delta\tau_{zx} = \frac{q}{\pi}\left[\sin\alpha \;\sin(\alpha + 2\beta)\right]$$

z/b	x/b								
	0			0.25			0.5		
	Ds$_z$/q	Ds$_x$/q	Dt$_{zx}$/q	Ds$_z$/q	Ds$_x$/q	Dt$_{zx}$/q	Ds$_z$/q	Ds$_x$/q	Dt$_{zx}$/q
0	1	1	0	1	1	0	0.5	0.5	0.32
0.1	0.9968	0.75191	0	0.98815	0.68523	0.03834	0.49979	0.43676	0.3152
0.25	0.9595	0.45018	0	0.90223	0.39294	0.12732	0.49692	0.34712	0.2996
0.35	0.9103	0.31212	0	0.83098	0.28592	0.15385	0.49208	0.29358	0.2836
0.5	0.8183	0.18169	0	0.73465	0.18618	0.15671	0.47974	0.22509	0.2546
0.75	0.6682	0.08051	0	0.60706	0.09777	0.12732	0.44796	0.14238	0.2037
1	0.5498	0.04052	0	0.5105	0.05513	0.09587	0.40915	0.09085	0.1592
1.25	0.4618	0.02271	0	0.4365	0.03321	0.07202	0.37005	0.0595	0.1242
1.5	0.3958	0.01385	0	0.37909	0.02121	0.05506	0.33408	0.04025	0.0979
1.75	0.3453	0.00901	0	0.33386	0.01423	0.04303	0.30237	0.02813	0.0784
2	0.3058	0.00617	0	0.29761	0.00996	0.03435	0.27491	0.02026	0.0637
3	0.2084	0.0019	0	0.20568	0.0032	0.01653	0.19791	0.00692	0.0318
4	0.1575	0.00081	0	0.15634	0.00139	0.00957	0.15288	0.00308	0.0187
5	0.1265	0.00042	0	0.12587	0.00072	0.00621	0.12405	0.00162	0.0122
6	0.1056	0.00024	0	0.10526	0.00042	0.00435	0.10419	0.00095	0.0086

2 Ứng suất

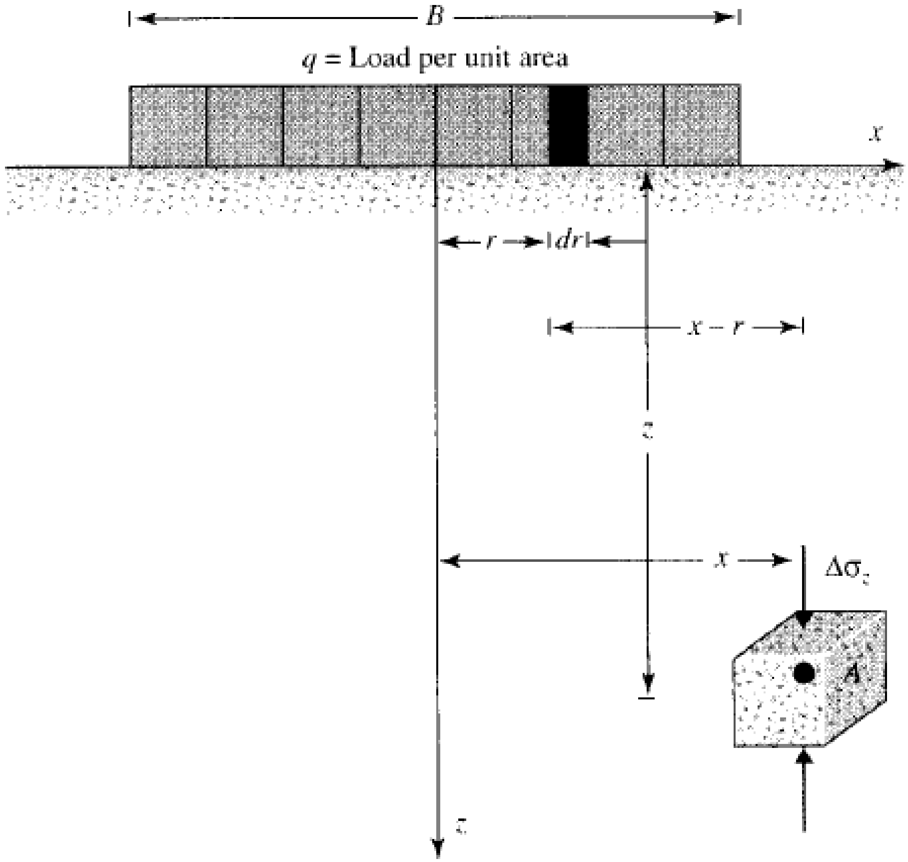
z/b	x/b								
	1			1.5			2		
	$\Delta\sigma_z/q$	$\Delta\sigma_x/q$	$\Delta\tau_{zx}/q$	$\Delta\sigma_z/q$	$\Delta\sigma_x/q$	$\Delta\tau_{zx}/q$	$\Delta\sigma_z/q$	$\Delta\sigma_x/q$	$\Delta\tau_{zx}/q$
0	0	0	0	0	0	0	0	0	0
0.1	0.00159	0.09227	0.01145	0.000196	0.037804	0.00264	5.5E-05	0.02111	0.00106
0.25	0.01985	0.19615	0.05876	0.002873	0.089634	0.01557	0.00083	0.05138	0.00641
0.35	0.04376	0.23475	0.09522	0.007348	0.118435	0.02862	0.0022	0.0698	0.01216
0.5	0.08776	0.25628	0.14043	0.01864	0.150861	0.05142	0.00597	0.09372	0.02323
0.75	0.15637	0.2409	0.18112	0.046879	0.17724	0.08831	0.01718	0.12203	0.04494
1	0.20483	0.20483	0.19099	0.079488	0.178274	0.11525	0.03333	0.13617	0.06611
1.25	0.23437	0.16777	0.18499	0.109691	0.165589	0.13043	0.05194	0.13901	0.08337
1.5	0.25005	0.13546	0.17189	0.134331	0.147294	0.13611	0.07059	0.13425	0.09549
1.75	0.25627	0.10894	0.15626	0.152774	0.127927	0.13528	0.0876	0.12509	0.10268
2	0.25628	0.08776	0.14043	0.165589	0.109691	0.13043	0.10212	0.11388	0.10578
3	0.22989	0.0393	0.08934	0.179779	0.057673	0.09862	0.13425	0.07059	0.09549
4	0.19615	0.01985	0.05876	0.167993	0.031664	0.07069	0.13877	0.04249	0.07535
5	0.16764	0.01115	0.04075	0.150861	0.01864	0.05142	0.13207	0.02642	0.05798
6	0.1451	0.0068	0.02964	0.134489	0.011706	0.03848	0.12203	0.01718	0.04494

2z/B	2x/B 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.000
0.10	1.000	1.000	0.999	0.999	0.999	0.998	0.997	0.993	0.980	0.909	0.500
0.20	0.997	0.997	0.996	0.995	0.992	0.988	0.979	0.959	0.909	0.775	0.500
0.30	0.990	0.989	0.987	0.984	0.978	0.967	0.947	0.908	0.833	0.697	0.499
0.40	0.977	0.976	0.973	0.966	0.955	0.937	0.906	0.855	0.773	0.651	0.498
0.50	0.959	0.958	0.953	0.943	0.927	0.902	0.864	0.808	0.727	0.620	0.497
0.60	0.937	0.935	0.928	0.915	0.896	0.866	0.825	0.767	0.691	0.598	0.495
0.70	0.910	0.908	0.899	0.885	0.863	0.831	0.788	0.732	0.662	0.581	0.492
0.80	0.881	0.878	0.869	0.853	0.829	0.797	0.755	0.701	0.638	0.566	0.489
0.90	0.850	0.847	0.837	0.821	0.797	0.765	0.724	0.675	0.617	0.552	0.485
1.00	0.818	0.815	0.805	0.789	0.766	0.735	0.696	0.650	0.598	0.540	0.480
1.10	0.787	0.783	0.774	0.758	0.735	0.706	0.670	0.628	0.580	0.529	0.474
1.20	0.755	0.752	0.743	0.728	0.707	0.679	0.646	0.607	0.564	0.517	0.468
1.30	0.725	0.722	0.714	0.699	0.679	0.654	0.623	0.588	0.548	0.506	0.462
1.40	0.696	0.693	0.685	0.672	0.653	0.630	0.602	0.569	0.534	0.495	0.455
1.50	0.668	0.666	0.658	0.646	0.629	0.607	0.581	0.552	0.519	0.484	0.448
1.60	0.642	0.639	0.633	0.621	0.605	0.586	0.562	0.535	0.506	0.474	0.440
1.70	0.617	0.615	0.608	0.598	0.583	0.565	0.544	0.519	0.492	0.463	0.433
1.80	0.593	0.591	0.585	0.576	0.563	0.546	0.526	0.504	0.479	0.453	0.425
1.90	0.571	0.569	0.564	0.555	0.543	0.528	0.510	0.489	0.467	0.443	0.417
2.00	0.550	0.548	0.543	0.535	0.524	0.510	0.494	0.475	0.455	0.433	0.409
2.10	0.530	0.529	0.524	0.517	0.507	0.494	0.479	0.462	0.443	0.423	0.401
2.20	0.511	0.510	0.506	0.499	0.490	0.479	0.465	0.449	0.432	0.413	0.393
2.30	0.494	0.493	0.489	0.483	0.474	0.464	0.451	0.437	0.421	0.404	0.385
2.40	0.477	0.476	0.473	0.467	0.460	0.450	0.438	0.425	0.410	0.395	0.378
2.50	0.462	0.461	0.458	0.452	0.445	0.436	0.426	0.414	0.400	0.386	0.370
2.60	0.447	0.446	0.443	0.439	0.432	0.424	0.414	0.403	0.390	0.377	0.363
2.70	0.433	0.432	0.430	0.425	0.419	0.412	0.403	0.393	0.381	0.369	0.355
2.80	0.420	0.419	0.417	0.413	0.407	0.400	0.392	0.383	0.372	0.360	0.348
2.90	0.408	0.407	0.405	0.401	0.396	0.389	0.382	0.373	0.363	0.352	0.341
3.00	0.396	0.395	0.393	0.390	0.385	0.379	0.372	0.364	0.355	0.345	0.334
3.10	0.385	0.384	0.382	0.379	0.375	0.369	0.363	0.355	0.347	0.337	0.327
3.20	0.374	0.373	0.372	0.369	0.365	0.360	0.354	0.347	0.339	0.330	0.321
3.30	0.364	0.363	0.362	0.359	0.355	0.351	0.345	0.339	0.331	0.323	0.315
3.40	0.354	0.354	0.352	0.350	0.346	0.342	0.337	0.331	0.324	0.316	0.308
3.50	0.345	0.345	0.343	0.341	0.338	0.334	0.329	0.323	0.317	0.310	0.302
3.60	0.337	0.336	0.335	0.333	0.330	0.326	0.321	0.316	0.310	0.304	0.297
3.70	0.328	0.328	0.327	0.325	0.322	0.318	0.314	0.309	0.304	0.298	0.291
3.80	0.320	0.320	0.319	0.317	0.315	0.311	0.307	0.303	0.297	0.292	0.285
3.90	0.313	0.313	0.312	0.310	0.307	0.304	0.301	0.296	0.291	0.286	0.280
4.00	0.306	0.305	0.304	0.303	0.301	0.298	0.294	0.290	0.285	0.280	0.275
4.10	0.299	0.299	0.298	0.296	0.294	0.291	0.288	0.284	0.280	0.275	0.270
4.20	0.292	0.292	0.291	0.290	0.288	0.285	0.282	0.278	0.274	0.270	0.265
4.30	0.286	0.286	0.285	0.283	0.282	0.279	0.276	0.273	0.269	0.265	0.260
4.40	0.280	0.280	0.279	0.278	0.276	0.274	0.271	0.268	0.264	0.260	0.256
4.50	0.274	0.274	0.273	0.272	0.270	0.268	0.266	0.263	0.259	0.255	0.251
4.60	0.268	0.268	0.268	0.266	0.265	0.263	0.260	0.258	0.254	0.251	0.247
4.70	0.263	0.263	0.262	0.261	0.260	0.258	0.255	0.253	0.250	0.246	0.243
4.80	0.258	0.258	0.257	0.256	0.255	0.253	0.251	0.248	0.245	0.242	0.239
4.90	0.253	0.253	0.252	0.251	0.250	0.248	0.246	0.244	0.241	0.238	0.235
5.00	0.248	0.248	0.247	0.246	0.245	0.244	0.242	0.239	0.237	0.234	0.231

2z/B	2x/B 1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0.00	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.10	0.091	0.020	0.007	0.003	0.002	0.001	0.001	0.000	0.000	0.000
0.20	0.225	0.091	0.040	0.020	0.011	0.007	0.004	0.003	0.002	0.002
0.30	0.301	0.165	0.090	0.052	0.031	0.020	0.013	0.009	0.007	0.005
0.40	0.346	0.224	0.141	0.090	0.059	0.040	0.027	0.020	0.014	0.011
0.50	0.373	0.267	0.185	0.128	0.089	0.063	0.046	0.034	0.025	0.019
0.60	0.391	0.298	0.222	0.163	0.120	0.088	0.066	0.050	0.038	0.030
0.70	0.403	0.321	0.250	0.193	0.148	0.113	0.087	0.068	0.053	0.042
0.80	0.411	0.338	0.273	0.218	0.173	0.137	0.108	0.086	0.069	0.056
0.90	0.416	0.351	0.291	0.239	0.195	0.158	0.128	0.104	0.085	0.070
1.00	0.419	0.360	0.305	0.256	0.214	0.177	0.147	0.122	0.101	0.084
1.10	0.420	0.366	0.316	0.271	0.230	0.194	0.164	0.138	0.116	0.098
1.20	0.419	0.371	0.325	0.282	0.243	0.209	0.178	0.152	0.130	0.111
1.30	0.417	0.373	0.331	0.291	0.254	0.221	0.191	0.166	0.143	0.123
1.40	0.414	0.374	0.335	0.298	0.263	0.232	0.203	0.177	0.155	0.135
1.50	0.411	0.374	0.338	0.303	0.271	0.240	0.213	0.188	0.165	0.146
1.60	0.407	0.373	0.339	0.307	0.276	0.248	0.221	0.197	0.175	0.155
1.70	0.402	0.370	0.339	0.309	0.281	0.254	0.228	0.205	0.183	0.164
1.80	0.396	0.368	0.339	0.311	0.284	0.258	0.234	0.212	0.191	0.172
1.90	0.391	0.364	0.338	0.312	0.286	0.262	0.239	0.217	0.197	0.179
2.00	0.385	0.360	0.336	0.311	0.288	0.265	0.243	0.222	0.203	0.185
2.10	0.379	0.356	0.333	0.311	0.288	0.267	0.246	0.226	0.208	0.190
2.20	0.373	0.352	0.330	0.309	0.288	0.268	0.248	0.229	0.212	0.195
2.30	0.366	0.347	0.327	0.307	0.288	0.268	0.250	0.232	0.215	0.199
2.40	0.360	0.342	0.323	0.305	0.287	0.268	0.251	0.234	0.217	0.202
2.50	0.354	0.337	0.320	0.302	0.285	0.268	0.251	0.235	0.220	0.205
2.60	0.347	0.332	0.316	0.299	0.283	0.267	0.251	0.236	0.221	0.207
2.70	0.341	0.327	0.312	0.296	0.281	0.266	0.251	0.236	0.222	0.208
2.80	0.335	0.321	0.307	0.293	0.279	0.265	0.250	0.236	0.223	0.210
2.90	0.329	0.316	0.303	0.290	0.276	0.263	0.249	0.236	0.223	0.211
3.00	0.323	0.311	0.299	0.286	0.274	0.261	0.248	0.236	0.223	0.211
3.10	0.317	0.306	0.294	0.283	0.271	0.259	0.247	0.235	0.223	0.212
3.20	0.311	0.301	0.290	0.279	0.268	0.256	0.245	0.234	0.223	0.212
3.30	0.305	0.296	0.286	0.275	0.265	0.254	0.243	0.232	0.222	0.211
3.40	0.300	0.291	0.281	0.271	0.261	0.251	0.241	0.231	0.221	0.211
3.50	0.294	0.286	0.277	0.268	0.258	0.249	0.239	0.229	0.220	0.210
3.60	0.289	0.281	0.273	0.264	0.255	0.246	0.237	0.228	0.218	0.209
3.70	0.284	0.276	0.268	0.260	0.252	0.243	0.235	0.226	0.217	0.208
3.80	0.279	0.272	0.264	0.256	0.249	0.240	0.232	0.224	0.216	0.207
3.90	0.274	0.267	0.260	0.253	0.245	0.238	0.230	0.222	0.214	0.206
4.00	0.269	0.263	0.256	0.249	0.242	0.235	0.227	0.220	0.212	0.205
4.10	0.264	0.258	0.252	0.246	0.239	0.232	0.225	0.218	0.211	0.203
4.20	0.260	0.254	0.248	0.242	0.236	0.229	0.222	0.216	0.209	0.202
4.30	0.255	0.250	0.244	0.239	0.233	0.226	0.220	0.213	0.207	0.200
4.40	0.251	0.246	0.241	0.235	0.229	0.224	0.217	0.211	0.205	0.199
4.50	0.247	0.242	0.237	0.232	0.226	0.221	0.215	0.209	0.203	0.197
4.60	0.243	0.238	0.234	0.229	0.223	0.218	0.212	0.207	0.201	0.195
4.70	0.239	0.235	0.230	0.225	0.220	0.215	0.210	0.205	0.199	0.194
4.80	0.235	0.231	0.227	0.222	0.217	0.213	0.208	0.202	0.197	0.192
4.90	0.231	0.227	0.223	0.219	0.215	0.210	0.205	0.200	0.195	0.190
5.00	0.227	0.224	0.220	0.216	0.212	0.207	0.203	0.198	0.193	0.188

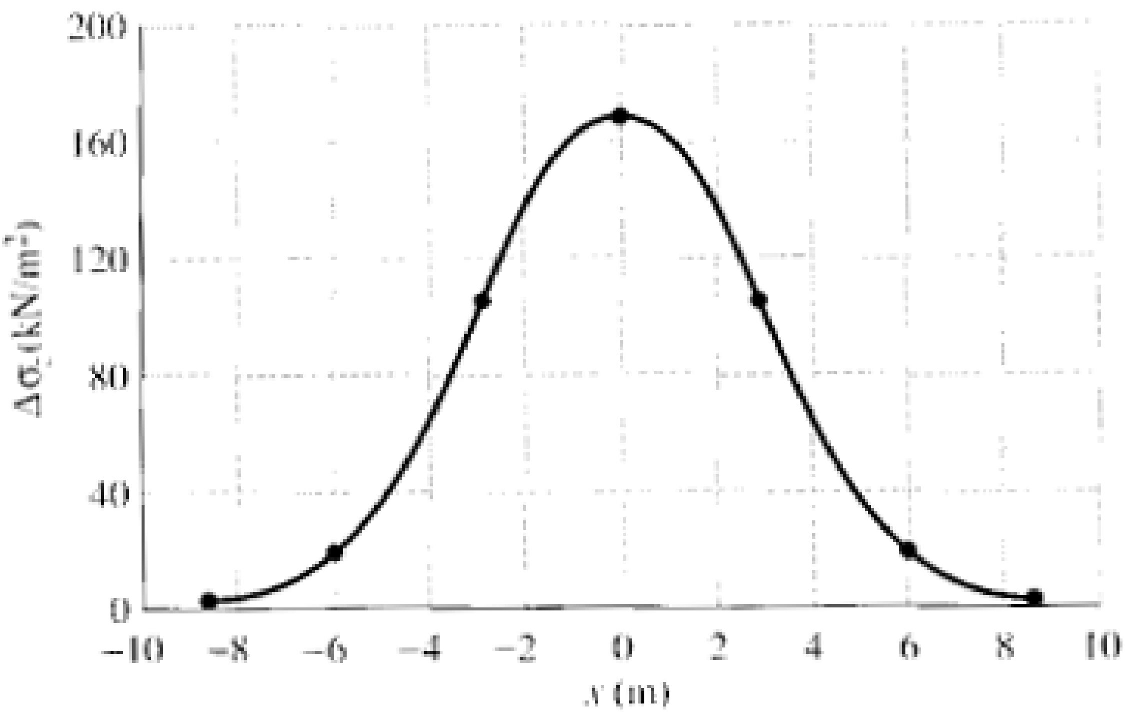
Thí dụ

With reference to Figure 9.8, we are given $q = 200$ kN/m^2, $B = 6$ m, and $z = 3$ m. Determine the vertical stress increase at $x = \pm 9, \pm 6, \pm 3$, and 0 m. Plot a graph of $\Delta\sigma_z$ against x.



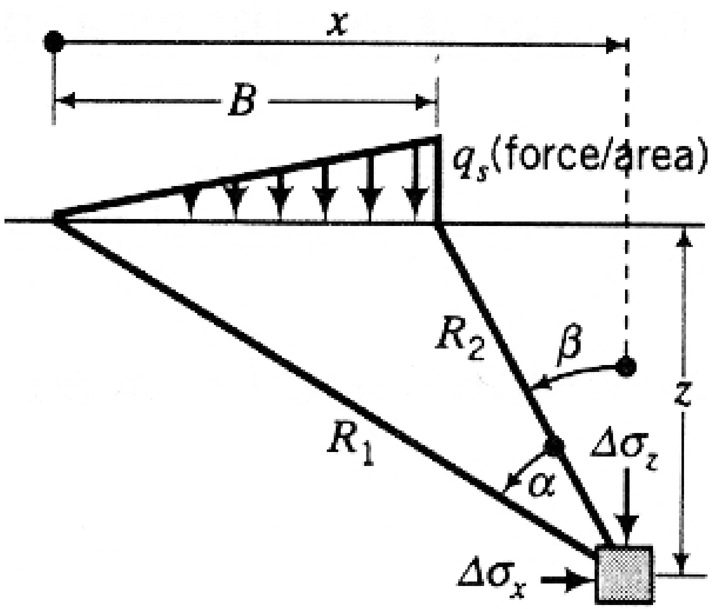
Solution

x(m)	2x/B	2z/B	$\Delta\sigma_z/q^a$	$\Delta\sigma_z{}^b$ (kN/m²)
±9	±3	1	0.017	3.4
±6	±2	1	0.084	16.8
±3	±1	1	0.480	96.0
0	0	1	0.818	163.6



2 Ứng suất

2.3.4 Ứng suất do tải trọng tam giác



$$\Delta s_z = \frac{q}{p}\left(\frac{x}{B}a - \frac{1}{2}\sin 2b\right)$$

$$\Delta s_x = \frac{q}{p}\left(\frac{x}{B}a - \frac{z}{B}\ln\frac{R_1^2}{R_2^2} + \frac{1}{2}\sin 2b\right)$$

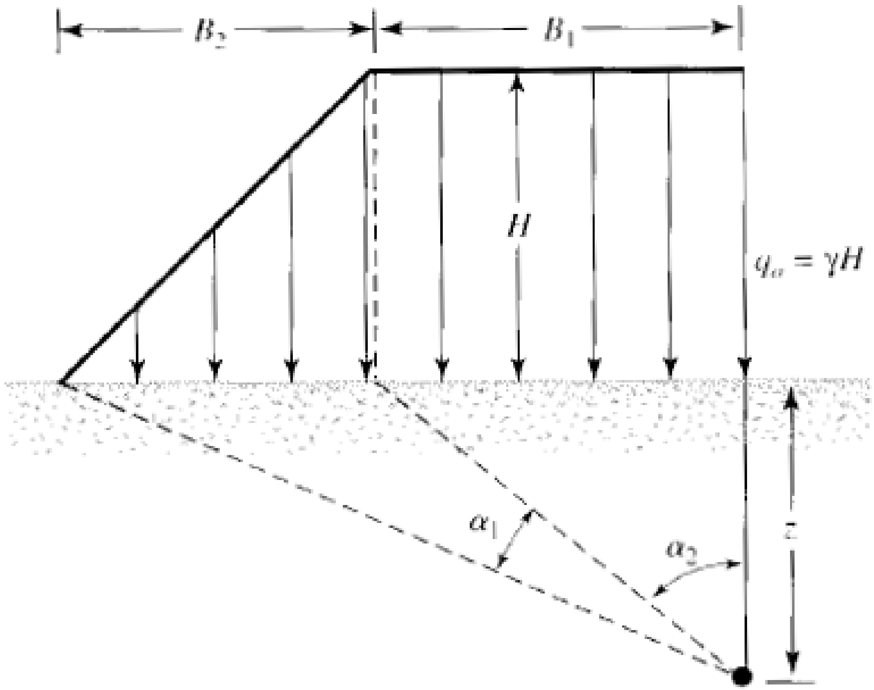
$$\Delta t_{zx} = \frac{q}{2p}\left(1 + \cos 2b - 2\frac{z}{B}a\right)$$

Values of σ_z/q

	z/b								
x/b	0	0.5	1.0	1.5	2.0	2.5	3.0	4.0	5.0
−3	0	0.0003	0.0018	0.00054	0.0107	0.0170	0.0235	0.0347	0.0422
−2	0	0.0008	0.0053	0.0140	0.0249	0.0356	0.0448	0.0567	0.0616
−1	0	0.0041	0.0217	0.0447	0.0643	0.0777	0.0854	0.0894	0.0858
0	0	0.0748	0.1273	0.1528	0.1592	0.1553	0.1469	0.1273	0.1098
1	0.5	0.4797	0.4092	0.3341	0.2749	0.2309	0.1979	0.1735	0.1241
2	0.5	0.4220	0.3524	0.2952	0.2500	0.2148	0.1872	0.1476	0.1211
3	0	0.0152	0.0622	0.1010	0.1206	0.1268	0.1258	0.1154	0.1026
4	0	0.0019	0.0119	0.0285	0.0457	0.0596	0.0691	0.0775	0.0776
5	0	0.0005	0.0035	0.0097	0.0182	0.0274	0.0358	0.0482	0.0546

2 Ứng suất

2.3.5 Ứng suất đứng do tải trọng nền đường



$$\Delta\sigma_z = \frac{q_o}{\pi}\left[\left(\frac{B_1 + B_2}{B_2}\right)(\alpha_1 + \alpha_2) - \frac{B_1}{B_2}(\alpha_2)\right]$$

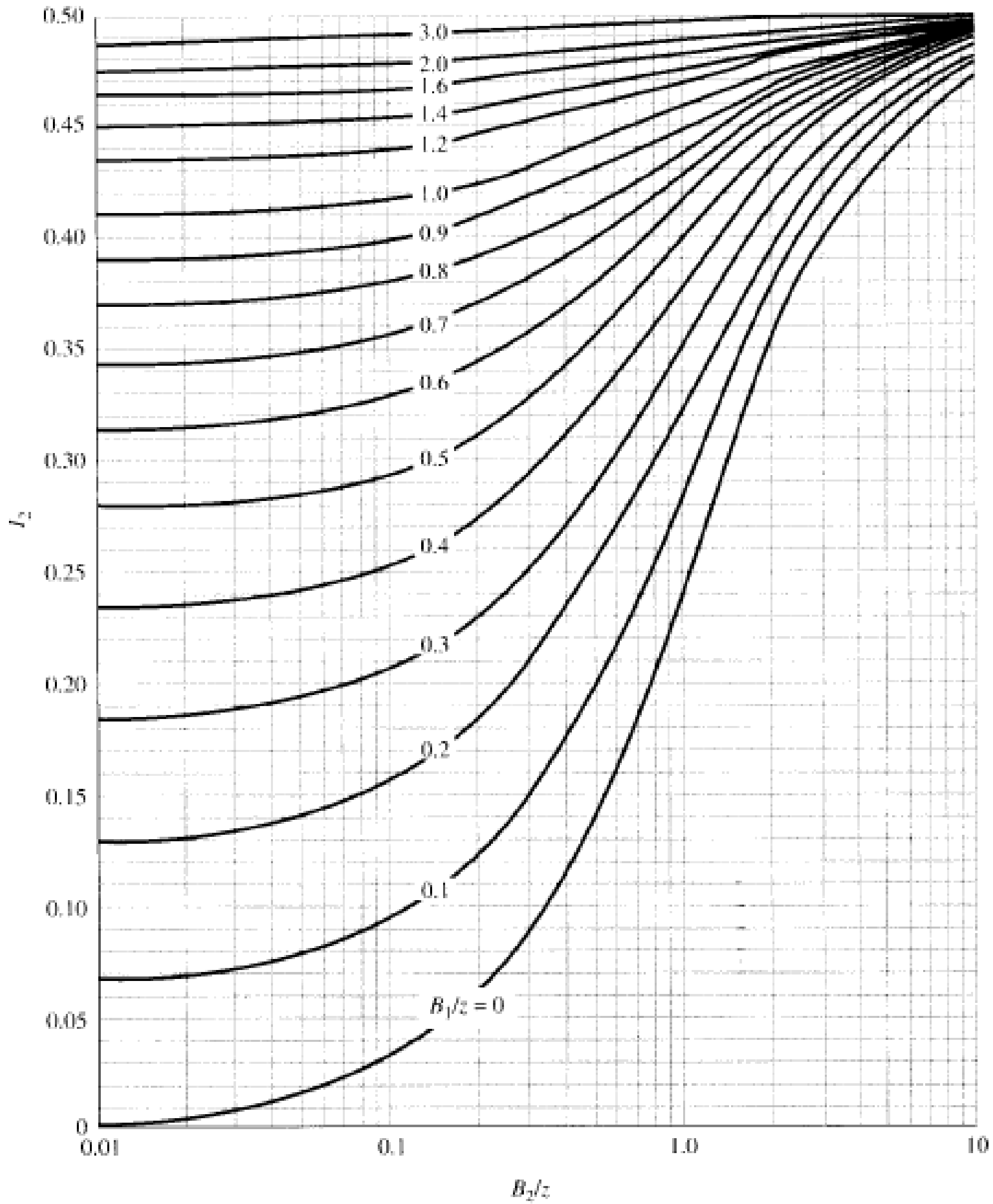
$$q_o = \gamma H$$

$$\alpha_1 \text{ (radians)} = \tan^{-1}\left(\frac{B_1 + B_2}{z}\right) - \tan^{-1}\left(\frac{B_1}{z}\right)$$

$$\alpha_2 = \tan^{-1}\left(\frac{B_1}{z}\right)$$

2 Ứng suất

$$\Delta s_z = q_o I_2$$



2 Ứng suất

- **Thí dụ**

An embankment is shown in Figure 9.12. Determine the stress increase under the embankment at points A_1 and A_2.

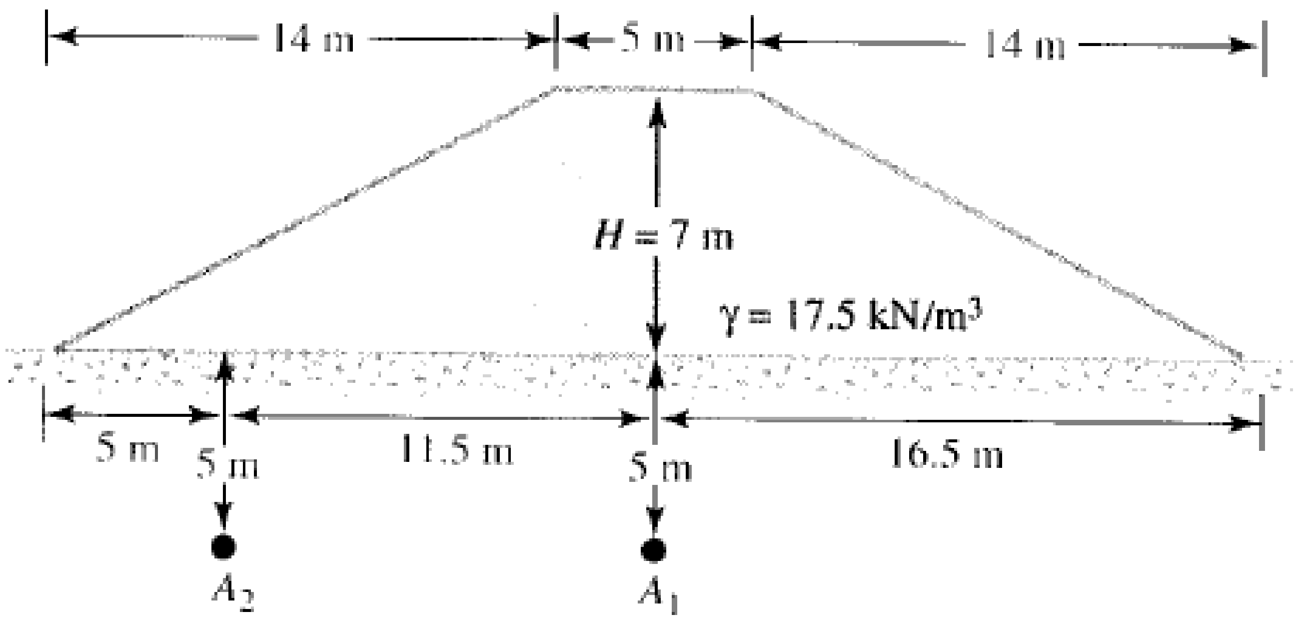
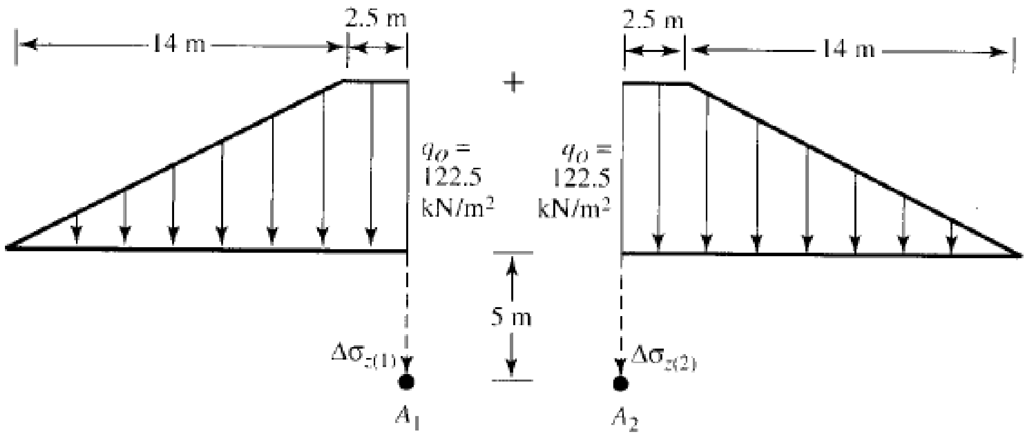


Figure 9.12

2 Ứng suất

Solution

Ở A_1



$$\gamma H = (17.5)(7) = 122.5 \text{ kN/m}^2$$

$$B_1 = 2.5 \text{ m and } B_2 = 14 \text{ m.}$$

$$\frac{B_1}{z} = \frac{2.5}{5} = 0.5; \frac{B_2}{z} = \frac{14}{5} = 2.8$$

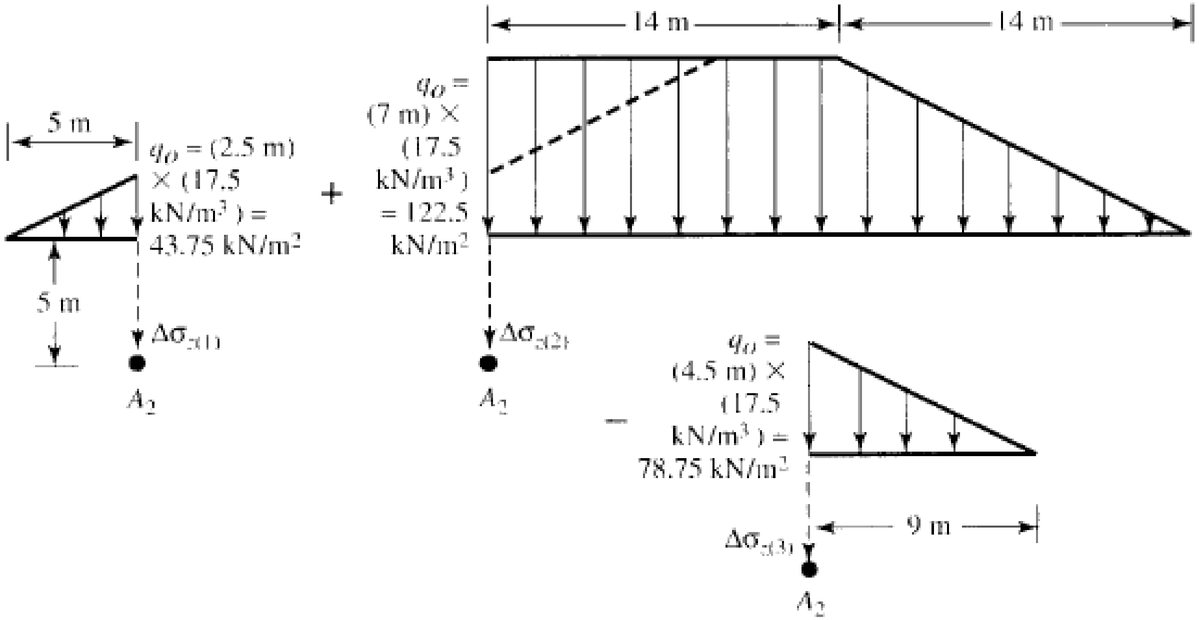
$$\rightarrow I_2 = 0.445$$

$$\Delta\sigma_z = \Delta\sigma_{z(1)} + \Delta\sigma_{z(2)} = q_o[I_{2(\text{Left})} + I_{2(\text{Right})}]$$

$$= 122.5[0.445 + 0.445] = \mathbf{109.03 \text{ kN/m}^2}$$

2 Ứng suất

Ở A_2



For the left side, $B_2 = 5$ m and $B_1 = 0$. So

$$\frac{B_2}{z} = \frac{5}{5} = 1; \quad \frac{B_1}{z} = \frac{0}{5} = 0$$

→ $I_2 = 0.25.$

$$\Delta\sigma_{z(1)} = 43.75(0.25) = 10.94 \text{ kN/m}^2$$

For the middle section,

$$\frac{B_2}{z} = \frac{14}{5} = 2.8; \quad \frac{B_1}{z} = \frac{14}{5} = 2.8$$

→ $I_2 = 0.495.$

$$\Delta\sigma_{z(2)} = 0.495(122.5) = 60.64 \text{ kN/m}^2$$

For the right side

$$\frac{B_2}{z} = \frac{9}{5} = 1.8; \quad \frac{B_1}{z} = \frac{0}{5} = 0$$

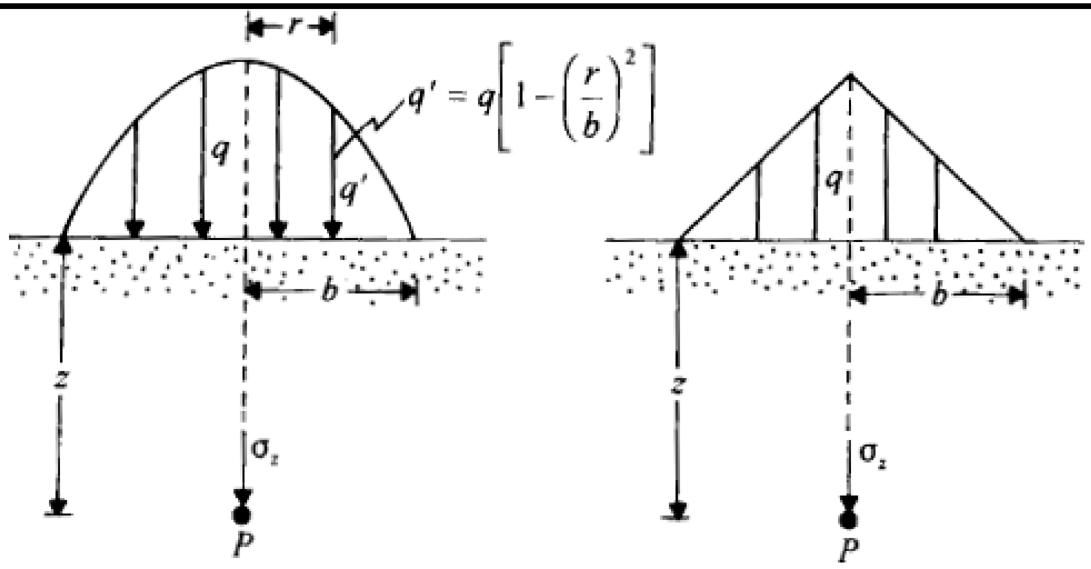
→ $I_2 = 0.335.$

$$\Delta\sigma_{z(3)} = (78.75)(0.335) = 26.38 \text{ kN/m}^2$$

Total stress increase at point A_2 is

$$\Delta\sigma_z = \Delta\sigma_{z(1)} + \Delta\sigma_{z(2)} - \Delta\sigma_{z(3)} = 10.94 + 60.64 - 26.38 = \mathbf{45.2\ kN/m^2}$$

2.3.6 Ứng suất đứng do tải trọng parabolic và nón



(a) (b)

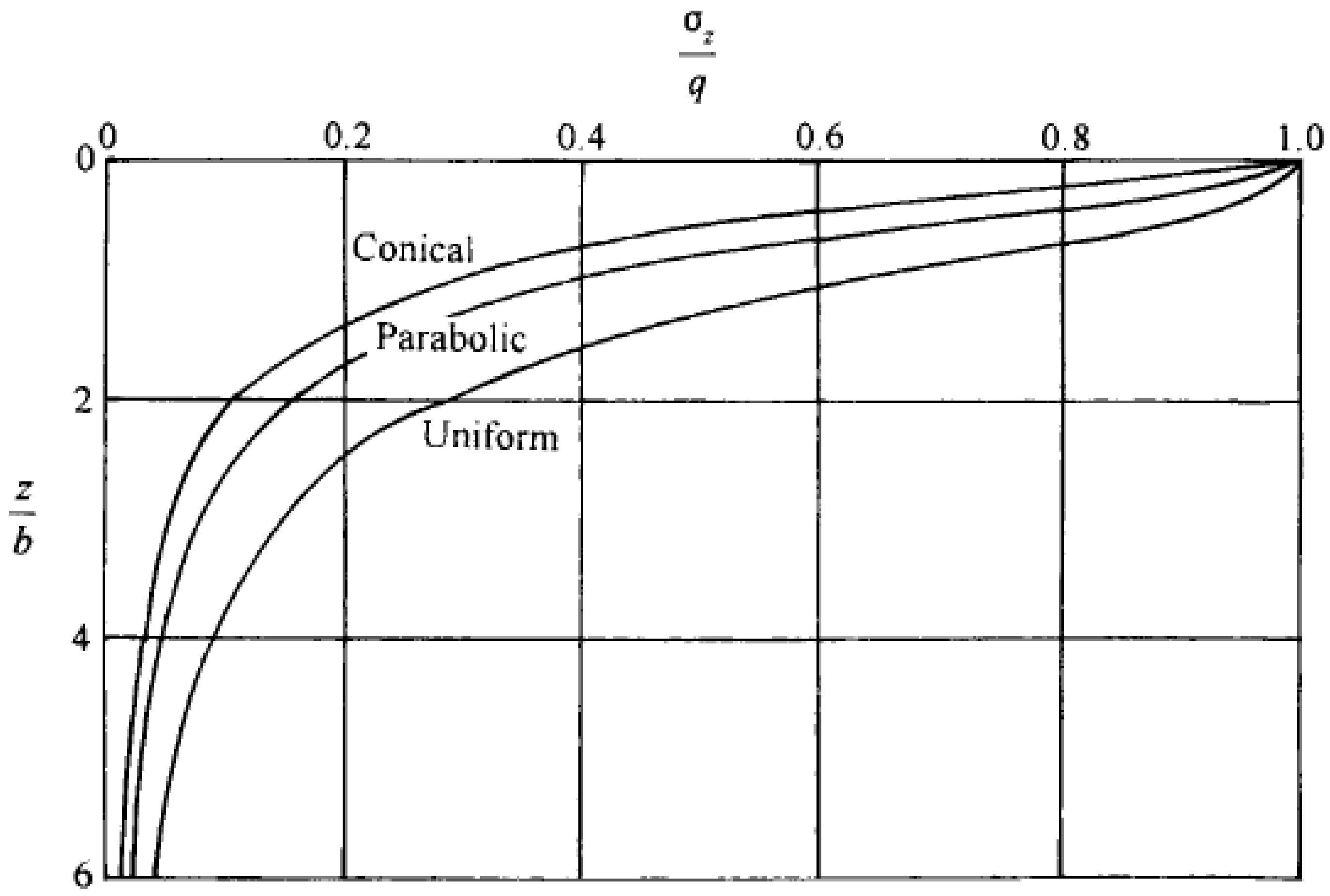
Parabolic loading

$$\frac{\sigma_z}{q} = \left[\frac{1}{(z/b) + \sqrt{1 + (z/b)^2}}\right]^2 \left[1 + \frac{2(z/b)}{\sqrt{1 + (z/b)^2}}\right]$$

Conical loading

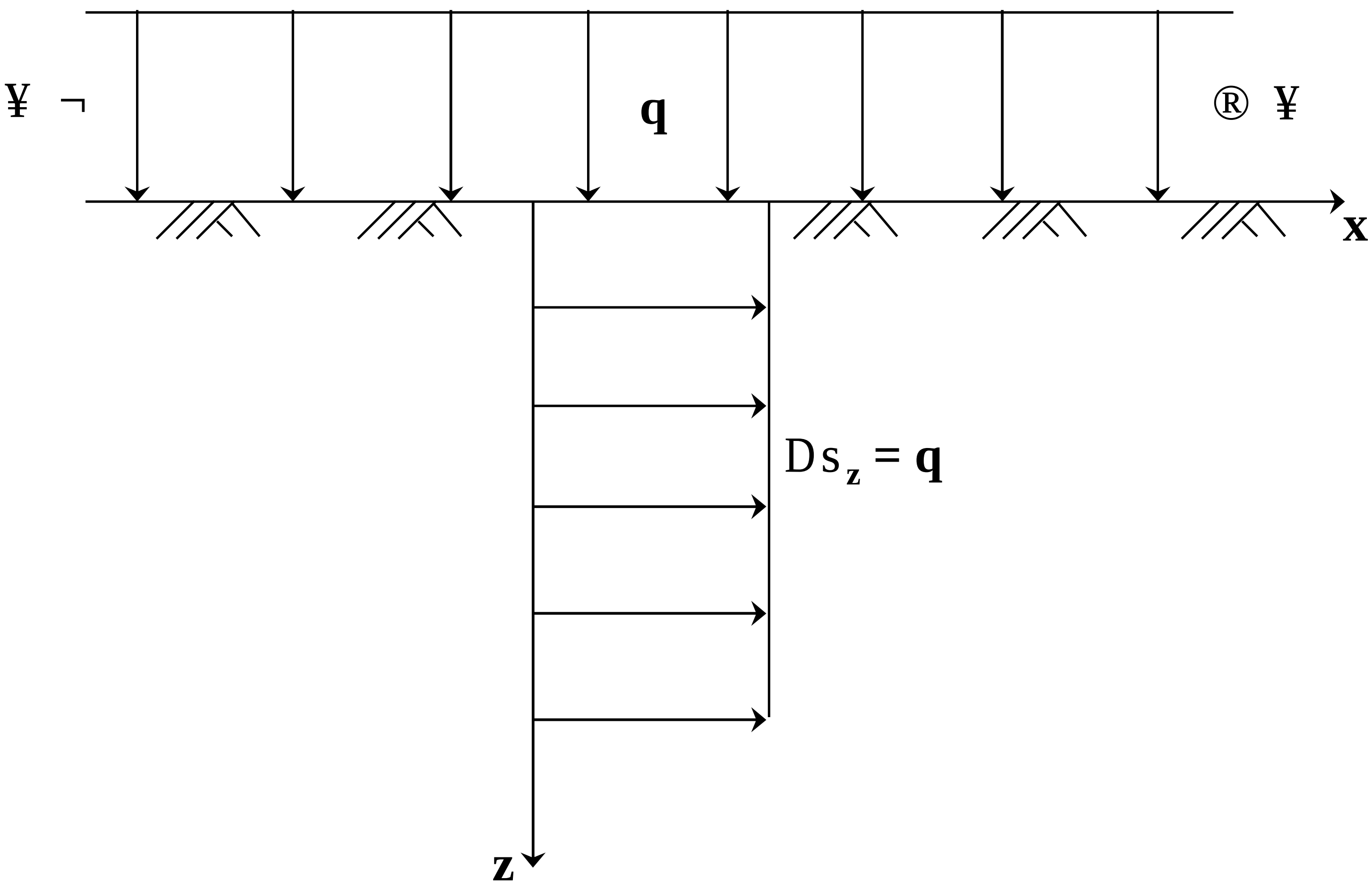
$$\frac{\sigma_z}{q} = \left\{1 - \frac{1}{[(b/z)^2 + 1]^{0.5}}\right\}$$

2 Ứng suất



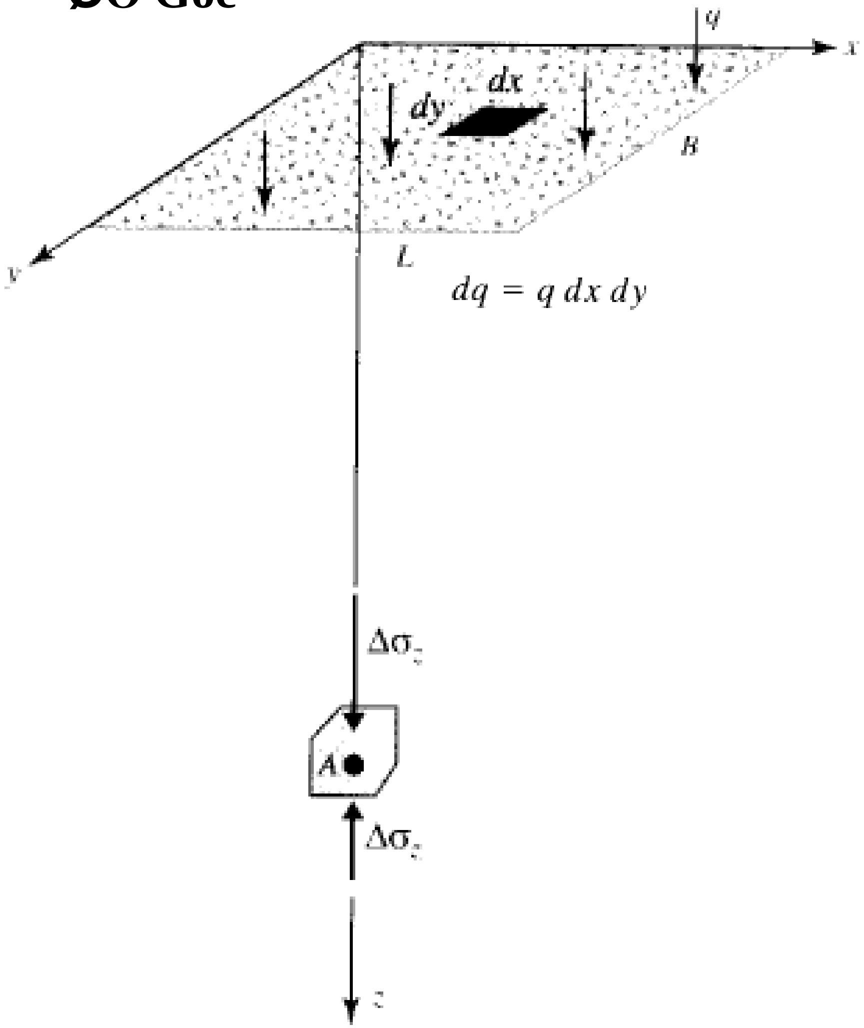
2 Ứng suất

2.3.7 Ứng suất do tải trọng đều khắp (một chiều)



2.3.8 Ứng suất do tải trọng đều phân bố trên diện tích chữ nhật

ØỞ Góc



$$d\sigma_z = \frac{3q\,dx\,dy\,z^3}{2\pi(x^2 + y^2 + z^2)^{5/2}}$$

$$\Delta\sigma_z = \int d\sigma_z = \int_{y=0}^{B}\int_{x=0}^{L} \frac{3qz^3(dx\,dy)}{2\pi(x^2 + y^2 + z^2)^{5/2}} = qI_3$$

where

$$I_3 = \frac{1}{4\pi}\left[\frac{2mn\sqrt{m^2 + n^2 + 1}}{m^2 + n^2 + m^2n^2 + 1}\left(\frac{m^2 + n^2 + 2}{m^2 + n^2 + 1}\right) + \tan^{-1}\left(\frac{2mn\sqrt{m^2 + n^2 + 1}}{m^2 + n^2 - m^2n^2 + 1}\right)\right]$$

$$m = \frac{B}{z} \qquad n = \frac{L}{z}$$

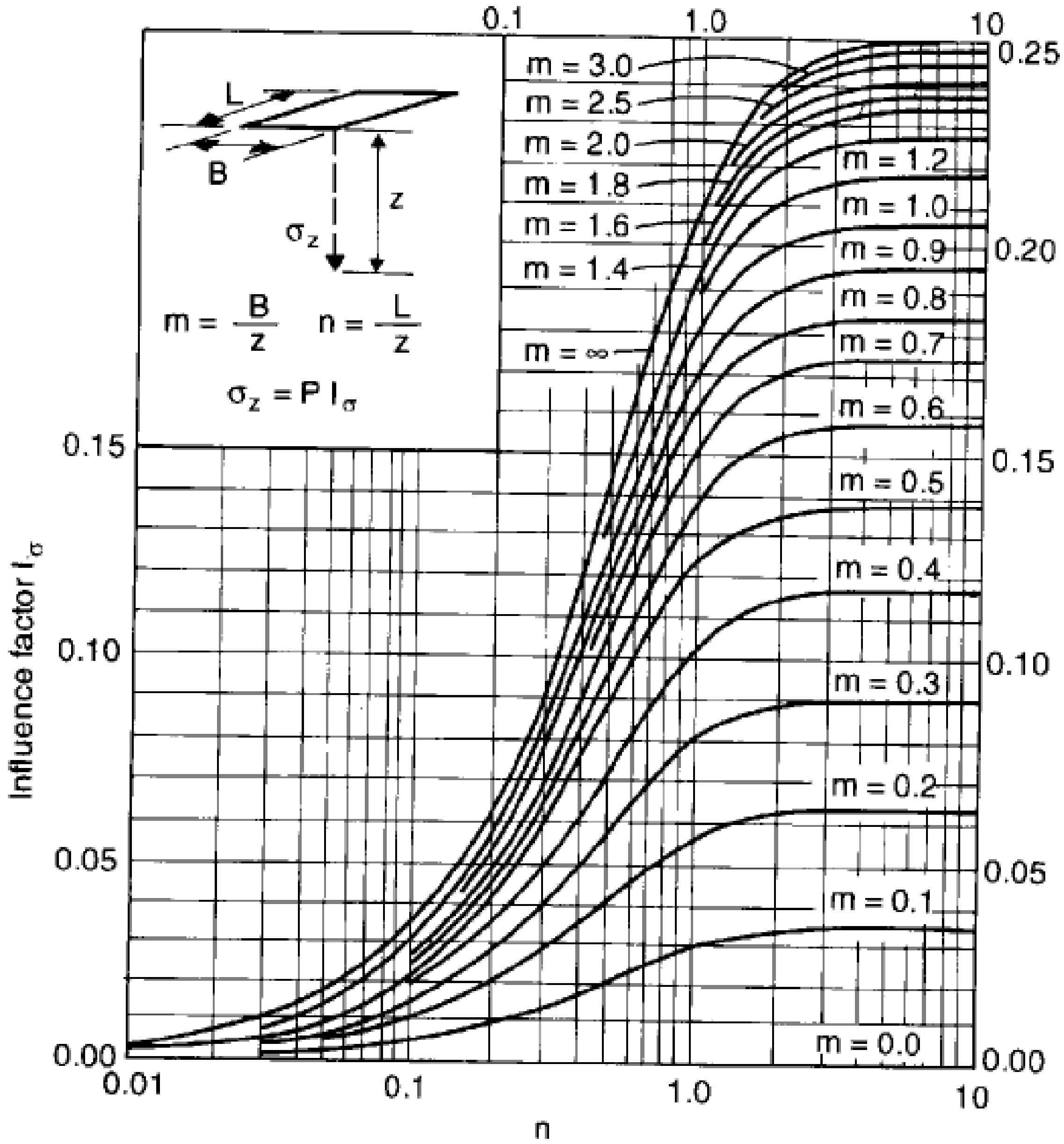
Variation of I_3 with m and n

	m									
n	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.1	0.0047	0.0092	0.0132	0.0168	0.0198	0.0222	0.0242	0.0258	0.0270	0.0279
0.2	0.0092	0.0179	0.0259	0.0328	0.0387	0.0435	0.0474	0.0504	0.0528	0.0547
0.3	0.0132	0.0259	0.0374	0.0474	0.0559	0.0629	0.0686	0.0731	0.0766	0.0794
0.4	0.0168	0.0328	0.0474	0.0602	0.0711	0.0801	0.0873	0.0931	0.0977	0.1013
0.5	0.0198	0.0387	0.0559	0.0711	0.0840	0.0947	0.1034	0.1104	0.1158	0.1202
0.6	0.0222	0.0435	0.0629	0.0801	0.0947	0.1069	0.1168	0.1247	0.1311	0.1361
0.7	0.0242	0.0474	0.0686	0.0873	0.1034	0.1169	0.1277	0.1365	0.1436	0.1491
0.8	0.0258	0.0504	0.0731	0.0931	0.1104	0.1247	0.1365	0.1461	0.1537	0.1598
0.9	0.0270	0.0528	0.0766	0.0977	0.1158	0.1311	0.1436	0.1537	0.1619	0.1684
1.0	0.0279	0.0547	0.0794	0.1013	0.1202	0.1361	0.1491	0.1598	0.1684	0.1752
1.2	0.0293	0.0573	0.0832	0.1063	0.1263	0.1431	0.1570	0.1684	0.1777	0.1851
1.4	0.0301	0.0589	0.0856	0.1094	0.1300	0.1475	0.1620	0.1739	0.1836	0.1914
1.6	0.0306	0.0599	0.0871	0.1114	0.1324	0.1503	0.1652	0.1774	0.1874	0.1955
1.8	0.0309	0.0606	0.0880	0.1126	0.1340	0.1521	0.1672	0.1797	0.1899	0.1981
2.0	0.0311	0.0610	0.0887	0.1134	0.1350	0.1533	0.1686	0.1812	0.1915	0.1999
2.5	0.0314	0.0616	0.0895	0.1145	0.1363	0.1548	0.1704	0.1832	0.1938	0.2024
3.0	0.0315	0.0618	0.0898	0.1150	0.1368	0.1555	0.1711	0.1841	0.1947	0.2034
4.0	0.0316	0.0619	0.0901	0.1153	0.1372	0.1560	0.1717	0.1847	0.1954	0.2042
5.0	0.0316	0.0620	0.0901	0.1154	0.1374	0.1561	0.1719	0.1849	0.1956	0.2044
6.0	0.0316	0.0620	0.0902	0.1154	0.1374	0.1562	0.1719	0.1850	0.1957	0.2045

2 Ứng suất

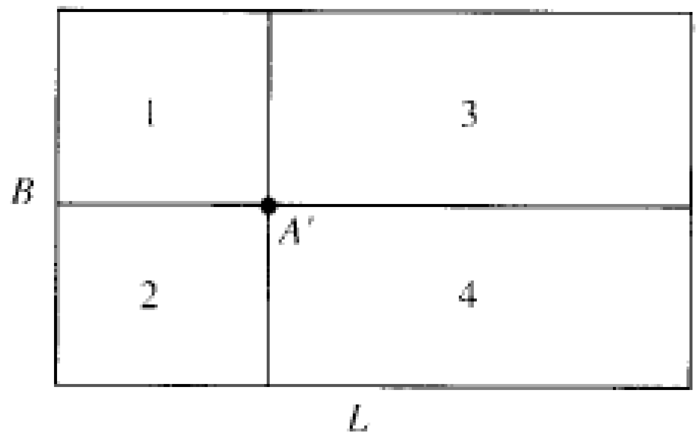
1.2	1.4	1.6	1.8	2.0	2.5	3.0	4.0	5.0	6.0
0.0293	0.0301	0.0306	0.0309	0.0311	0.0314	0.0315	0.0316	0.0316	0.0316
0.0573	0.0589	0.0599	0.0606	0.0610	0.0616	0.0618	0.0619	0.0620	0.0620
0.0832	0.0856	0.0871	0.0880	0.0887	0.0895	0.0898	0.0901	0.0901	0.0902
0.1063	0.1094	0.1114	0.1126	0.1134	0.1145	0.1150	0.1153	0.1154	0.1154
0.1263	0.1300	0.1324	0.1340	0.1350	0.1363	0.1368	0.1372	0.1374	0.1374
0.1431	0.1475	0.1503	0.1521	0.1533	0.1548	0.1555	0.1560	0.1561	0.1562
0.1570	0.1620	0.1652	0.1672	0.1686	0.1704	0.1711	0.1717	0.1719	0.1719
0.1684	0.1739	0.1774	0.1797	0.1812	0.1832	0.1841	0.1847	0.1849	0.1850
0.1777	0.1836	0.1874	0.1899	0.1915	0.1938	0.1947	0.1954	0.1956	0.1957
0.1851	0.1914	0.1955	0.1981	0.1999	0.2024	0.2034	0.2042	0.2044	0.2045
0.1958	0.2028	0.2073	0.2103	0.2124	0.2151	0.2163	0.2172	0.2175	0.2176
0.2028	0.2102	0.2151	0.2184	0.2206	0.2236	0.2250	0.2260	0.2263	0.2264
0.2073	0.2151	0.2203	0.2237	0.2261	0.2294	0.2309	0.2320	0.2323	0.2325
0.2103	0.2183	0.2237	0.2274	0.2299	0.2333	0.2350	0.2362	0.2366	0.2367
0.2124	0.2206	0.2261	0.2299	0.2325	0.2361	0.2378	0.2391	0.2395	0.2397
0.2151	0.2236	0.2294	0.2333	0.2361	0.2401	0.2420	0.2434	0.2439	0.2441
0.2163	0.2250	0.2309	0.2350	0.2378	0.2420	0.2439	0.2455	0.2461	0.2463
0.2172	0.2260	0.2320	0.2362	0.2391	0.2434	0.2455	0.2472	0.2479	0.2481
0.2175	0.2263	0.2324	0.2366	0.2395	0.2439	0.2460	0.2479	0.2486	0.2489
0.2176	0.2264	0.2325	0.2367	0.2397	0.2441	0.2463	0.2482	0.2489	0.2492

2 Ứng suất



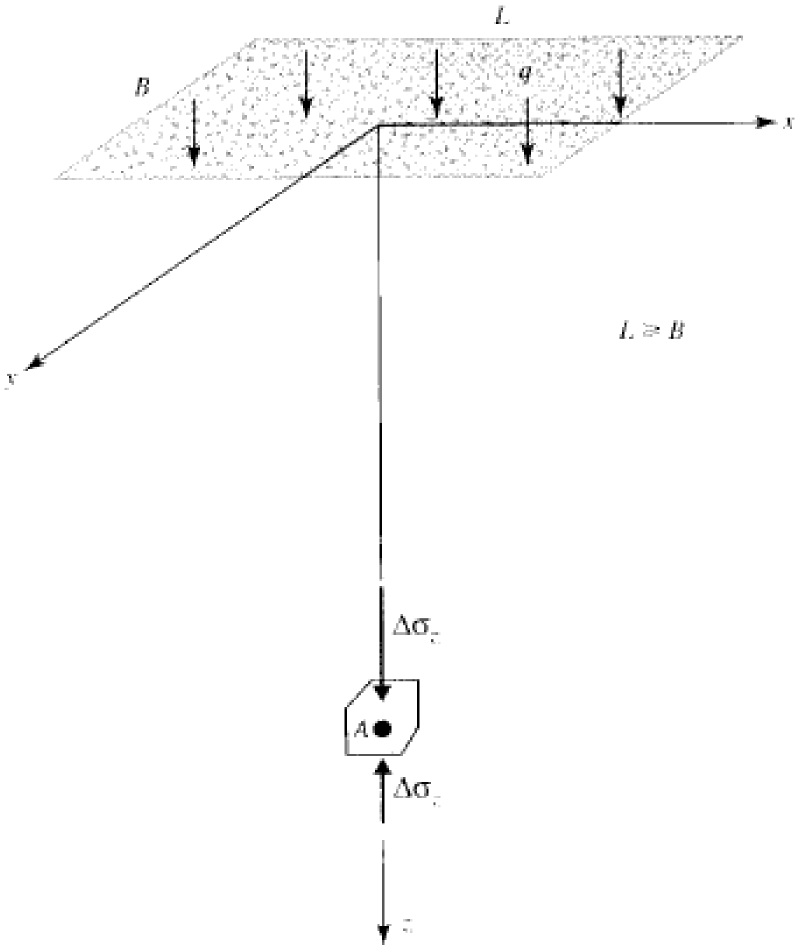
L
B
z
σz
m = B/z
n = L/z
σz = P Iσ
m = 3.0
m = 2.5
m = 2.0
m = 1.8
m = 1.6
m = 1.4
m = ∞
m = 1.2
m = 1.0
m = 0.9
m = 0.8
m = 0.7
m = 0.6
m = 0.5
m = 0.4
m = 0.3
m = 0.2
m = 0.1
m = 0.0
Influence factor Iσ
0.00
0.05
0.10
0.15
0.20
0.25
0.01
0.1
1.0
10
n

2 Ứng suất



$$\Delta\sigma_z = q[I_{3(1)} + I_{3(2)} + I_{3(3)} + I_{3(4)}]$$

Ø Ở Tâm



$$\Delta\sigma_z = qI_4$$

$$I_4 = \frac{2}{\pi}\left[\frac{m_1 n_1}{\sqrt{1+m_1^2+n_1^2}}\,\frac{1+m_1^2+2n_1^2}{(1+n_1^2)(m_1^2+n_1^2)}\right.$$

$$\left. + \sin^{-1}\frac{m_1}{\sqrt{m_1^2+n_1^2}\sqrt{1+n_1^2}}\right]$$

$$m_1 = \frac{L}{B} \qquad n_1 = \frac{z}{b} \qquad b = \frac{B}{2}$$

Variation of I_4 with m_1 and n_1

n_1	m_1 = 1	2	3	4	5	6	7	8	9	10
0.20	0.994	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997	0.997
0.40	0.960	0.976	0.977	0.977	0.977	0.977	0.977	0.977	0.977	0.977
0.60	0.892	0.932	0.936	0.936	0.937	0.937	0.937	0.937	0.937	0.937
0.80	0.800	0.870	0.878	0.880	0.881	0.881	0.881	0.881	0.881	0.881
1.00	0.701	0.800	0.814	0.817	0.818	0.818	0.818	0.818	0.818	0.818
1.20	0.606	0.727	0.748	0.753	0.754	0.755	0.755	0.755	0.755	0.755
1.40	0.522	0.658	0.685	0.692	0.694	0.695	0.695	0.696	0.696	0.696
1.60	0.449	0.593	0.627	0.636	0.639	0.640	0.641	0.641	0.641	0.642
1.80	0.388	0.534	0.573	0.585	0.590	0.591	0.592	0.592	0.593	0.593
2.00	0.336	0.481	0.525	0.540	0.545	0.547	0.548	0.549	0.549	0.549
3.00	0.179	0.293	0.348	0.373	0.384	0.389	0.392	0.393	0.394	0.395
4.00	0.108	0.190	0.241	0.269	0.285	0.293	0.298	0.301	0.302	0.303
5.00	0.072	0.131	0.174	0.202	0.219	0.229	0.236	0.240	0.242	0.244
6.00	0.051	0.095	0.130	0.155	0.172	0.184	0.192	0.197	0.200	0.202
7.00	0.038	0.072	0.100	0.122	0.139	0.150	0.158	0.164	0.168	0.171
8.00	0.029	0.056	0.079	0.098	0.113	0.125	0.133	0.139	0.144	0.147
9.00	0.023	0.045	0.064	0.081	0.094	0.105	0.113	0.119	0.124	0.128
10.00	0.019	0.037	0.053	0.067	0.079	0.089	0.097	0.103	0.108	0.112

2 Ứng suất

$$\Delta s_{x} = \frac{q}{2p}\left(\tan^{-1}\frac{LB}{zR_3} - \frac{LBz}{R_1^2 R_3}\right)$$

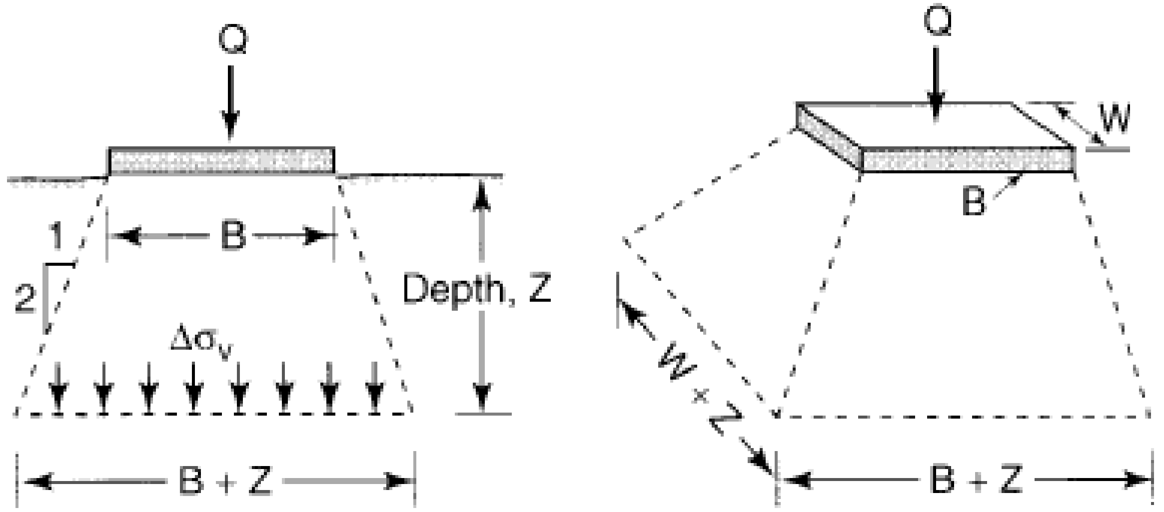
$$\Delta s_{y} = \frac{q}{2p}\left(\tan^{-1}\frac{LB}{zR_3} - \frac{LBz}{R_2^2 R_3}\right)$$

$$\Delta t_{zx} = \frac{q}{2p}\left(\frac{B}{R_2} - \frac{z^2 B}{R_1^2 R_3}\right)$$

$$R_1 = \sqrt{L^2 + z^2} \qquad R_2 = \sqrt{B^2 + z^2} \qquad R_3 = \sqrt{L^2 + B^2 + z^2}$$

2 Ứng suất

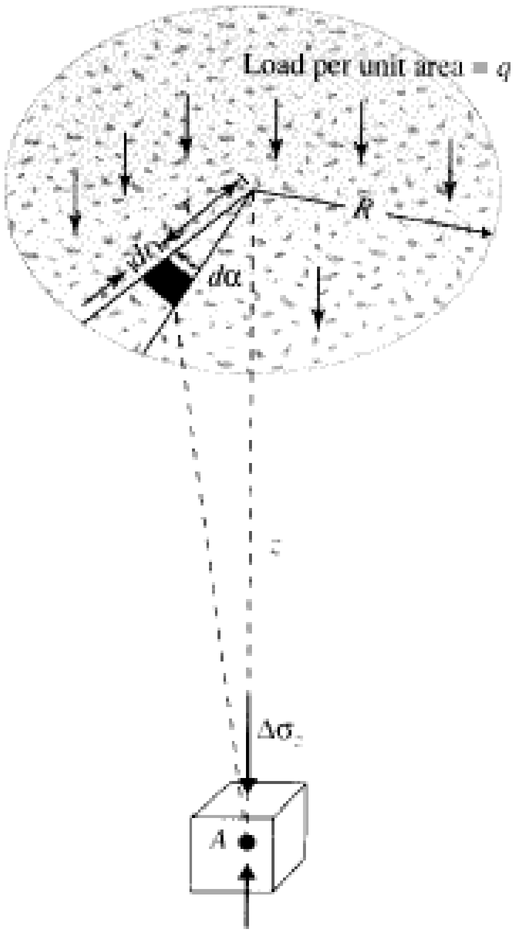
Ø Phương pháp gần đúng xác định ứng suất thẳng đứng



$$\Delta\sigma_v = \frac{Q}{(B+Z)(W+Z)}$$

2.3.9 Ứng suất do tải trọng đều phân bố trên diện tích hình tròn

- **Bên dưới tâm của tải trọng tác dụng**



$$d\sigma_z = \frac{3(qr\,dr\,d\alpha)}{2\pi}\frac{z^3}{(r^2+z^2)^{5/2}}$$

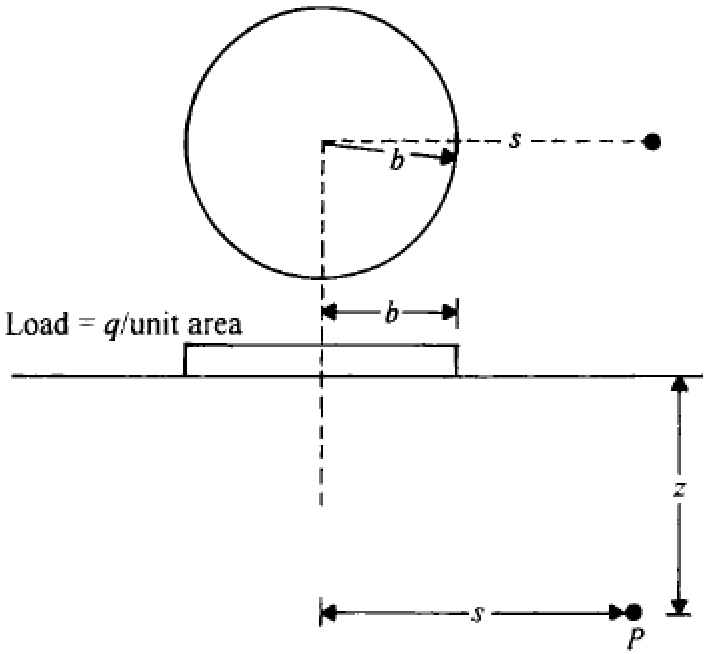
$$\Delta\sigma_z = \int d\sigma_z = \int_{\alpha=0}^{\alpha=2\pi}\int_{r=0}^{r=R}\frac{3q}{2\pi}\frac{z^3 r}{(r^2+z^2)^{5/2}}\,dr\,d\alpha$$

$$\Delta\sigma_z = q\left\{1-\frac{1}{[(R/z)^2+1]^{3/2}}\right\}$$

$$\Delta s_r = \frac{q}{2}\left[(1+2n)-\frac{2(1+n)}{\left[1+(R/z)^2\right]^{1/2}}+\frac{1}{\left[1+(R/z)^2\right]^{3/2}}\right]$$

2 Ứng suất

- Ở điểm bất kỳ



$$\sigma_z = q(A' + B')$$

$$\sigma_r = q[2\nu A' + C + (1 - 2\nu)F]$$

$$\sigma_\theta = q[2\nu A' - D + (1 - 2\nu)E]$$

$$\tau_{rz} = \tau_{zr} = qG$$

$$\sigma_P = \frac{(\sigma_z + \sigma_r) \pm \sqrt{(\sigma_z - \sigma_r)^2 + (2\tau_{rz})^2}}{2}$$

Function A'

z/b	s/b: 0	0.2	0.4	0.6	0.8	1	1.2	1.5	2
0	1.0	1.0	1.0	1.0	1.0	.5	0	0	0
0.1	.90050	.89748	.88679	.86126	.78797	.43015	.09645	.02787	.00856
0.2	.80388	.79824	.77884	.73483	.63014	.38269	.15433	.05251	.01680
0.3	.71265	.70518	.68316	.62690	.52081	.34375	.17964	.07199	.02440
0.4	.62861	.62015	.59241	.53767	.44329	.31048	.18709	.08593	.03118
0.5	.55279	.54403	.51622	.46448	.38390	.28156	.18556	.09499	.03701
0.6	.48550	.47691	.45078	.40427	.33676	.25588	.17952	.10010	
0.7	.42654	.41874	.39491	.35428	.29833	.21727	.17124	.10228	.04558
0.8	.37531	.36832	.34729	.31243	.26581	.21297	.16206	.10236	
0.9	.33104	.32492	.30669	.27707	.23832	.19488	.15253	.10094	
1	.29289	.28763	.27005	.24697	.21468	.17868	.14329	.09849	.05185
1.2	.23178	.22795	.21662	.19890	.17626	.15101	.12570	.09192	.05260
1.5	.16795	.16552	.15877	.14804	.13436	.11892	.10296	.08048	.05116
2	.10557	.10453	.10140	.09647	.09011	.08269	.07471	.06275	.04496
2.5	.07152	.07098	.06947	.06698	.06373	.05974	.05555	.04880	.03787
3	.05132	.05101	.05022	.04886	.04707	.04487	.04241	.03839	.03150
4	.02986	.02976	.02907	.02802	.02832	.02749	.02651	.02490	.02193
5	.01942	.01938				.01835			.01573
6	.01361					.01307			.01168
7	.01005					.00976			.00894
8	.00772					.00755			.00703
9	.00612					.00600			.00566
10								.00477	.00465

z/b	s/b = 3	4	5	6	7	8	10	12	14
0	0	0	0	0	0	0	0	0	0
0.1	.00211	.00084	.00042						
0.2	.00419	.00167	.00083	.00048	.00030	.00020			
0.3	.00622	.00250							
0.4									
0.5	.01013	.00407	.00209	.00118	.00071	.00053	.00025	.00014	.00009
0.6									
0.7									
0.8									
0.9									
1	.01742	.00761	.00393	.00226	.00143	.00097	.00050	.00029	.00018
1.2	.01935	.00871	.00459	.00269	.00171	.00115			
1.5	.02142	.01013	.00548	.00325	.00210	.00141	.00073	.00043	.00027
2	.02221	.01160	.00659	.00399	.00264	.00180	.00094	.00056	.00036
2.5	.02143	.01221	.00732	.00463	.00308	.00214	.00115	.00068	.00043
3	.01980	.01220	.00770	.00505	.00346	.00242	.00132	.00079	.00051
4	.01592	.01109	.00768	.00536	.00384	.00282	.00160	.00099	.00065
5	.01249	.00949	.00708	.00527	.00394	.00298	.00179	.00113	.00075
6	.00983	.00795	.00628	.00492	.00384	.00299	.00188	.00124	.00084
7	.00784	.00661	.00548	.00445	.00360	.00291	.00193	.00130	.00091
8	.00635	.00554	.00472	.00398	.00332	.00276	.00189	.00134	.00094
9	.00520	.00466	.00409	.00353	.00301	.00256	.00184	.00133	.00096
10	.00438	.00397	.00352	.00326	.00273	.00241			

After Ahlvin and Ulery (1962).

2 Ứng suất

Function *B′*

z/b	s/b: 0	0.2	0.4	0.6	0.8	1	1.2	1.5	2
0	0	0	0	0	0	0	0	0	0
0.1	.09852	.10140	.11138	.13424	.18796	.05388	−.07899	−.02672	−.00845
0.2	.18857	.19306	.20772	.23524	.25983	.08513	−.07759	−.04448	−.01593
0.3	.26362	.26787	.28018	.29483	.27257	.10757	−.04316	−.04999	−.02166
0.4	.32016	.32259	.32748	.32273	.26925	.12404	−.00766	−.04535	−.02522
0.5	.35777	.35752	.35323	.33106	.26236	.13591	.02165	−.03455	−.02651
0.6	.37831	.37531	.36308	.32822	.25411	.14440	.04457	−.02101	
0.7	.38487	.37962	.36072	.31929	.24638	.14986	.06209	−.00702	−.02329
0.8	.38091	.37408	.35133	.30699	.23779	.15292	.07530	.00614	
0.9	.36962	.36275	.33734	.29299	.22891	.15404	.08507	.01795	
1	.35355	.34553	.32075	.27819	.21978	.15355	.09210	.02814	−.01005
1.2	.31485	.30730	.28481	.24836	.20113	.14915	.10002	.04378	.00023
1.5	.25602	.25025	.23338	.20694	.17368	.13732	.10193	.05745	.01385
2	.17889	.18144	.16644	.15198	.13375	.11331	.09254	.06371	.02836
2.5	.12807	.12633	.12126	.11327	.10298	.09130	.07869	.06022	.03429
3	.09487	.09394	.09099	.08635	.08033	.07325	.06551	.05354	.03511
4	.05707	.05666	.05562	.05383	.05145	.04773	.04532	.03995	.03066
5	.03772	.03760				.03384			.02474
6	.02666					.02468			.01968
7	.01980					.01868			.01577
8	.01526					.01459			.01279
9	.01212					.01170			.01054
10								.00924	.00879

	s/b								
z/b	3	4	5	6	7	8	10	12	14
0	0	0	0	0	0	0	0	0	0
0.1	−.00210	−.00084	−.00042						
0.2	−.00412	−.00166	−.00083	−.00024	−.00015	−.00010			
0.3	−.00599	−.00245							
0.4									
0.5	−.00991	−.00388	−.00199	−.00116	−.00073	−.00049	−.00025	−.00014	−.00009
0.6									
0.7									
0.8									
0.9									
1	−.01115	−.00608	−.00344	−.00210	−.00135	−.00092	−.00048	−.00028	−.00018
1.2	−.00995	−.00632	−.00378	−.00236	−.00156	−.00107			
1.5	−.00669	−.00600	−.00401	−.00265	−.00181	−.00126	−.00068	−.00040	−.00026
2	.00028	−.00410	−.00371	−.00278	−.00202	−.00148	−.00084	−.00050	−.00033
2.5	.00661	−.00130	−.00271	−.00250	−.00201	−.00156	−.00094	−.00059	−.00039
3	.01112	.00157	−.00134	−.00192	−.00179	−.00151	−.00099	−.00065	−.00046
4	.01515	.00595	.00155	−.00029	−.00094	−.00109	−.00094	−.00068	−.00050
5	.01522	.00810	.00371	.00132	.00013	−.00043	−.00070	−.00061	−.00049
6	.01380	.00867	.00496	.00254	.00110	.00028	−.00037	−.00047	−.00045
7	.01204	.00842	.00547	.00332	.00185	.00093	−.00002	−.00029	−.00037
8	.01034	.00779	.00554	.00372	.00236	.00141	.00035	−.00008	−.00025
9	.00888	.00705	.00533	.00386	.00265	.00178	.00066	.00012	−.00012
10	.00764	.00631	.00501	.00382	.00281	.00199			

After Ahlvin and Ulery (1962).

Function *C*

z/b	s/b = 0	0.2	0.4	0.6	0.8	1	1.2	1.5	2
0	0	0	0	0	0	0	0	0	0
0.1	−.04926	−.05142	−.05903	−.07708	−.12108	.02247	.12007	.04475	.01536
0.2	−.09429	−.09775	−.10872	−.12977	−.14552	.02419	.14896	.07892	.02951
0.3	−.13181	−.13484	−.14415	−.15023	−.12990	.01988	.13394	.09816	.04148
0.4	−.16008	−.16188	−.16519	−.15985	−.11168	.01292	.11014	.10422	.05067
0.5	−.17889	−.17835	−.17497	−.15625	−.09833	.00483	.08730	.10125	.05690
0.6	−.18915	−.18664	−.17336	−.14934	−.08967	−.00304	.06731	.09313	
0.7	−.19244	−.18831	−.17393	−.14147	−.08409	−.01061	.05028	.08253	.06129
0.8	−.19046	−.18481	−.16784	−.13393	−.08066	−.01744	.03582	.07114	
0.9	−.18481	−.17841	−.16024	−.12664	−.07828	−.02337	.02359	.05993	
1	−.17678	−.17050	−.15188	−.11995	−.07634	−.02843	.01331	.04939	.05429
1.2	−.15742	−.15117	−.13467	−.10763	−.07289	−.03575	−.00245	.03107	.04552
1.5	−.12801	−.12277	−.11101	−.09145	−.06711	−.04124	−.01702	.01088	.03154
2	−.08944	−.08491	−.07976	−.06925	−.05560	−.04144	−.02687	−.00782	.01267
2.5	−.06403	−.06068	−.05839	−.05259	−.04522	−.03605	−.02800	−.01536	.00103
3	−.04744	−.04560	−.04339	−.04089	−.03642	−.03130	−.02587	−.01748	−.00528
4	−.02854	−.02737	−.02562	−.02585	−.02421	−.02112	−.01964	−.01586	−.00956
5	−.01886	−.01810				−.01568			−.00939
6	−.01333					−.01118			−.00819
7	−.00990					−.00902			−.00678
8	−.00763					−.00699			−.00552
9	−.00607					−.00423			−.00452
10								−.00381	−.00373

	s/b								
z/b	3	4	5	6	7	8	10	12	14
0	0	0	0	0	0	0	0	0	0
0.1	.00403	.00164	.00082						
0.2	.00796	.00325	.00164	.00094	.00059	.00039			
0.3	.01169	.00483							
0.4									
0.5	.01824	.00778	.00399	.00231	.00146	.00098	.00050	.00029	.00018
0.6									
0.7									
0.8									
0.9									
1	.02726	.01333	.00726	.00433	.00278	.00188	.00098	.00057	.00036
1.2	.02791	.01467	.00824	.00501	.00324	.00221			
1.5	.02652	.01570	.00933	.00585	.00386	.00266	.00141	.00083	.00039
2	.02070	.01527	.01013	.00321	.00462	.00327	.00179	.00107	.00069
2.5	.01384	.01314	.00987	.00707	.00506	.00369	.00209	.00128	.00083
3	.00792	.01030	.00888	.00689	.00520	.00392	.00232	.00145	.00096
4	.00038	.00492	.00602	.00561	.00476	.00389	.00254	.00168	.00115
5	−.00293	−.00128	.00329	.00391	.00380	.00341	.00250	.00177	.00127
6	−.00405	−.00079	.00129	.00234	.00272	.00272	.00227	.00173	.00130
7	−.00417	−.00180	−.00004	.00113	.00174	.00200	.00193	.00161	.00128
8	−.00393	−.00225	−.00077	.00029	.00096	.00134	.00157	.00143	.00120
9	−.00353	−.00235	−.00118	−.00027	.00037	.00082	.00124	.00122	.00110
10	−.00314	−.00233	−.00137	−.00063	.00030	.00040			

After Ahlvin and Ulery (1962).

Function *D*

z/b	s/b: 0	0.2	0.4	0.6	0.8	1	1.2	1.5	2
0	0	0	0	0	0	0	0	0	0
0.1	.04926	.04998	.05235	.05716	.06687	.07635	.04108	.01803	.00691
0.2	.09429	.09552	.09900	.10546	.11431	.10932	.07139	.03444	.01359
0.3	.13181	.13305	.14051	.14062	.14267	.12745	.09078	.04817	.01982
0.4	.16008	.16070	.16229	.16288	.15756	.13696	.10248	.05887	.02545
0.5	.17889	.17917	.17826	.17481	.16403	.14074	.10894	.06670	.03039
0.6	.18915	.18867	.18573	.17887	.16489	.14137	.11186	.07212	
0.7	.19244	.19132	.18679	.17782	.16229	.13926	.11237	.07551	.03801
0.8	.19046	.18927	.18348	.17306	.15714	.13548	.11115	.07728	
0.9	.18481	.18349	.17709	.16635	.15063	.13067	.10866	.07788	
1	.17678	.17503	.16886	.15824	.14344	.12513	.10540	.07753	.04456
1.2	.15742	.15618	.15014	.14073	.12823	.11340	.09757	.07484	.04575
1.5	.12801	.12754	.12237	.11549	.10657	.09608	.08491	.06833	.04539
2	.08944	.09080	.08668	.08273	.07814	.07187	.06566	.05589	.04103
2.5	.06403	.06565	.06284	.06068	.05777	.05525	.05069	.04486	.03532
3	.04744	.04834	.04760	.04548	.04391	.04195	.03963	.03606	.02983
4	.02854	.02928	.02996	.02798	.02724	.02661	.02568	.02408	.02110
5	.01886	.01950				.01816			.01535
6	.01333					.01351			.01149
7	.00990					.00966			.00899
8	.00763					.00759			.00727
9	.00607					.00746			.00601
10								.00542	.00506

	s/b								
z/b	3	4	5	6	7	8	10	12	14
0	0	0	0	0	0	0	0	0	0
0.1	.00193	.00080	.00041						
0.2	.00384	.00159	.00081	.00047	.00029	.00020			
0.3	.00927	.00238							
0.4									
0.5	.00921	.00390	.00200	.00116	.00073	.00049	.00025	.00015	.00009
0.6									
0.7									
0.8									
0.9									
1	.01611	.00725	.00382	.00224	.00142	.00096	.00050	.00029	.00018
1.2	.01796	.00835	.00446	.00264	.00169	.00114			
1.5	.01983	.00970	.00532	.00320	.00205	.00140	.00073	.00043	.00027
2	.02098	.01117	.00643	.00398	.00260	.00179	.00095	.00056	.00036
2.5	.02045	.01183	.00717	.00457	.00306	.00213	.00115	.00068	.00044
3	.01904	.01187	.00755	.00497	.00341	.00242	.00133	.00080	.00052
4	.01552	.01087	.00757	.00533	.00382	.00280	.00160	.00100	.00065
5	.01230	.00939	.00700	.00523	.00392	.00299	.00180	.00114	.00077
6	.00976	.00788	.00625	.00488	.00381	.00301	.00190	.00124	.00086
7	.00787	.00662	.00542	.00445	.00360	.00292	.00192	.00130	.00092
8	.00641	.00554	.00477	.00402	.00332	.00275	.00192	.00131	.00096
9	.00533	.00470	.00415	.00358	.00303	.00260	.00187	.00133	.00099
10	.00450	.00398	.00364	.00319	.00278	.00239			

After Ahlvin and Ulery (1962).

Function *E*

	s/b								
z/b	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2
0	.5	.5	.5	.5	.5	.5	.34722	.22222	.12500
0.1	.45025	.449494	.44698	.44173	.43008	.39198	.30445	.20399	.11806
0.2	.40194	.400434	.39591	.38660	.36798	.32802	.26598	.18633	.11121
0.3	.35633	.35428	.33809	.33674	.31578	.28003	.23311	.16967	.10450
0.4	.31431	.31214	.30541	.29298	.27243	.24200	.20526	.15428	.09801
0.5	.27639	.27407	.26732	.25511	.23639	.21119	.18168	.14028	.09180
0.6	.24275	.24247	.23411	.22289	.20634	.18520	.16155	.12759	
0.7	.21327	.21112	.20535	.19525	.18093	.16356	.14421	.11620	.08027
0.8	.18765	.18550	.18049	.17190	.15977	.14523	.12928	.10602	
0.9	.16552	.16337	.15921	.15179	.14168	.12954	.11634	.09686	
1	.14645	.14483	.14610	.13472	.12618	.11611	.10510	.08865	.06552
1.2	.11589	.11435	.11201	.10741	.10140	.09431	.08657	.07476	.05728
1.5	.08398	.08356	.08159	.07885	.07517	.07088	.06611	.05871	.04703
2	.05279	.05105	.05146	.05034	.04850	.04675	.04442	.04078	.03454
2.5	.03576	.03426	.03489	.03435	.03360	.03211	.03150	.02953	.02599
3	.02566	.02519	.02470	.02491	.02444	.02389	.02330	.02216	.02007
4	.01493	.01452	.01495	.01526	.01446	.01418	.01395	.01356	.01281
5	.00971	.00927				.00929			.00873
6	.00680					.00632			.00629
7	.00503					.00493			.00466
8	.00386					.00377			.00354
9	.00306					.00227			.00275
10								.00210	.00220

	s/b								
z/b	3	4	5	6	7	8	10	12	14
0	.05556	.03125	.02000	.01389	.01020	.00781	.00500	.00347	.00255
0.1	.05362	.03045	.01959						
0.2	.05170	.02965	.01919	.01342	.00991	.00762			
0.3	.04979	.02886							
0.4									
0.5	.04608	.02727	.01800	.01272	.00946	.00734	.00475	.00332	.00246
0.6									
0.7									
0.8									
0.9									
1	.03736	.02352	.01602	.01157	.00874	.00683	.00450	.00318	.00237
1.2	.03425	.02208	.01527	.01113	.00847	.00664			
1.5	.03003	.02008	.01419	.01049	.00806	.00636	.00425	.00304	.00228
2	.02410	.01706	.01248	.00943	.00738	.00590	.00401	.00290	.00219
2.5	.01945	.01447	.01096	.00850	.00674	.00546	.00378	.00276	.00210
3	.01585	.01230	.00962	.00763	.00617	.00505	.00355	.00263	.00201
4	.01084	.00900	.00742	.00612	.00511	.00431	.00313	.00237	.00185
5	.00774	.00673	.00579	.00495	.00425	.00364	.00275	.00213	.00168
6	.00574	.00517	.00457	.00404	.00354	.00309	.00241	.00192	.00154
7	.00438	.00404	.00370	.00330	.00296	.00264	.00213	.00172	.00140
8	.00344	.00325	.00297	.00273	.00250	.00228	.00185	.00155	.00127
9	.00273	.00264	.00246	.00229	.00212	.00194	.00163	.00139	.00116
10	.00225	.00221	.00203	.00200	.00181	.00171			

After Ahlvin and Ulery (1962).

Function *F*

	s/b								
z/b	0	0.2	0.4	0.6	0.8	1	1.2	1.5	2
0	.5	.5	.5	.5	.5	0	−.34722	−.22222	−.12500
0.1	.45025	.44794	.43981	.41954	.35789	.03817	−.20800	−.17612	−.10950
0.2	.40194	.39781	.38294	.34823	.26215	.05466	−.11165	−.13381	−.09441
0.3	.35633	.35094	.34508	.29016	.20503	.06372	−.05346	−.09768	−.08010
0.4	.31431	.30801	.28681	.24469	.17086	.06848	−.01818	−.06835	−.06684
0.5	.27639	.26997	.24890	.20937	.14752	.07037	.00388	−.04529	−.05479
0.6	.24275	.23444	.21667	.18138	.13042	.07068	.01797	−.02749	
0.7	.21327	.20762	.18956	.15903	.11740	.06963	.02704	−.01392	−.03469
0.8	.18765	.18287	.16679	.14053	.10604	.06774	.03277	−.00365	
0.9	.16552	.16158	.14747	.12528	.09664	.06533	.03619	.00408	
1	.14645	.14280	.12395	.11225	.08850	.06256	.03819	.00984	−.01367
1.2	.11589	.11360	.10460	.09449	.07486	.05670	.03913	.01716	−.00452
1.5	.08398	.08196	.07719	.06918	.05919	.04804	.03686	.02177	.00413
2	.05279	.05348	.04994	.04614	.04162	.03593	.03029	.02197	.01043
2.5	.03576	.03673	.03459	.03263	.03014	.02762	.02406	.01927	.01188
3	.02566	.02586	.02255	.02395	.02263	.02097	.01911	−.01623	.01144
4	.01493	.01536	.01412	.01259	.01386	.01331	.01256	.01134	.00912
5	.00971	.01011				.00905			.00700
6	.00680					.00675			.00538
7	.00503					.00483			.00428
8	.00386					.00380			.00350
9	.00306					.00374			.00291
10								.00267	.00246

	s/b								
z/b	3	4	5	6	7	8	10	12	14
0	−.05556	−.03125	−.02000	−.01389	−.01020	−.00781	−.00500	−.00347	−.00255
0.1	−.05151	−.02961	−.01917						
0.2	−.04750	−.02798	−.01835	−.01295	−.00961	−.00742			
0.3	−.04356	−.02636							
0.4									
0.5	−.03595	−.02320	−.01590	−.01154	−.00875	−.00681	−.00450	−.00318	−.00237
1	−.01994	−.01591	−.01209	−.00931	−.00731	−.00587	−.00400	−.00289	−.00219
1.2	−.01491	−.01337	−.01068	−.00844	−.00676	−.00550			
1.5	−.00879	−.00995	−.00870	−.00723	−.00596	−.00495	−.00353	−.00261	−.00201
2	−.00189	−.00546	−.00589	−.00544	−.00474	−.00410	−.00307	−.00233	−.00183
2.5	.00198	−.00226	−.00364	−.00386	−.00366	−.00332	−.00263	−.00208	−.00166
3	.00396	−.00010	−.00192	−.00258	−.00271	−.00263	−.00223	−.00183	−.00150
4	.00508	.00209	.00026	−.00076	−.00127	−.00148	−.00153	−.00137	−.00120
5	.00475	.00277	.00129	.00031	−.00030	−.00066	−.00096	−.00099	−.00093
6	.00409	.00278	.00170	.00088	.00030	−.00010	−.00053	−.00066	−.00070
7	.00346	.00258	.00178	.00114	.00064	.00027	−.00020	−.00041	−.00049
8	.00291	.00229	.00174	.00125	.00082	.00048	.00003	−.00020	−.00033
9	.00247	.00203	.00163	.00124	.00089	.00062	.00020	−.00005	−.00019
10	.00213	.00176	.00149	.00126	.00092	.00070			

After Ahlvin and Ulery (1962).

Function *G*

z/b	s/b = 0	0.2	0.4	0.6	0.8	1	1.2	1.5	2
0	0	0	0	0	0	.31831	0	0	0
0.1	0	.00315	.00802	.01951	.06682	.31405	.05555	.00865	.00159
0.2	0	.01163	.02877	.06441	.16214	.30474	.13592	.03060	.00614
0.3	0	.02301	.05475	.11072	.21465	.29228	.18216	.05747	.01302
0.4	0	.03460	.07883	.14477	.23442	.27779	.20195	.08233	.02138
0.5	0	.04429	.09618	.16426	.23652	.26216	.20731	.10185	.03033
0.6	0	.04966	.10729	.17192	.22949	.24574	.20496	.11541	
0.7	0	.05484	.11256	.17126	.21772	.22924	.19840	.12373	.04718
0.8	0	.05590	.11225	.16534	.20381	.21295	.18953	.12855	
0.9	0	.05496	.10856	.15628	.18904	.19712	.17945	.28881	
1	0	.05266	.10274	.14566	.17419	.18198	.16884	.12745	.06434
1.2	0	.04585	.08831	.12323	.14615	.15408	.14755	.12038	.06967
1.5	0	.03483	.06688	.09293	.11071	.11904	.11830	.10477	.07075
2	0	.02102	.04069	.05721	.06948	.07738	.08067	.07804	.06275
2.5	0	.01293	.02534	.03611	.04484	.05119	.05509	.05668	.05117
3	0	.00840	.01638	.02376	.02994	.03485	.03843	.04124	.04039
4	0	.00382	.00772	.01149	.01480	.01764	.02004	.02271	.02475
5	0	.00214				.00992		.01343	.01551
6	0					.00602		.00845	.01014
7	0					.00396			.00687
8	0					.00270			.00481
9	0					.00177			.00347
10	0							.00199	.00258

	s/b								
z/b	3	4	5	6	7	8	10	12	14
0	0	0	0	0	0	0	0	0	0
0.1	.00023	.00007	.00003						
0.2	.00091	.00026	.00010	.00005	.00003	.00002			
0.3	.00201	.00059							
0.4									
0.5	.00528	.00158	.00063	.00030	.00016	.00009	.00004	.00002	.00001
0.6									
0.7									
0.8									
0.9									
1	.01646	.00555	.00233	.00113	.00062	.00036	.00015	.00007	.00004
1.2	.02077	.00743	.00320	.00159	.00087	.00051			
1.5	.02599	.01021	.00460	.00233	.00130	.00078	.00033	.00016	.00009
2	.03062	.01409	.00692	.00369	.00212	.00129	.00055	.00027	.00015
2.5	.03099	.01650	.00886	.00499	.00296	.00185	.00082	.00041	.00023
3	.02886	.01745	.01022	.00610	.00376	.00241	.00110	.00057	.00032
4	.02215	.01639	.01118	.00745	.00499	.00340	.00167	.00090	.00052
5	.01601	.01364	.01105	.00782	.00560	.00404	.00216	.00122	.00073
6	.01148	.01082	.00917	.00733	.00567	.00432	.00243	.00150	.00092
7	.00830	.00842	.00770	.00656	.00539	.00432	.00272	.00171	.00110
8	.00612	.00656	.00631	.00568	.00492	.00413	.00278	.00185	.00124
9	.00459	.00513	.00515	.00485	.00438	.00381	.00274	.00192	.00133
10	.00351	.00407	.00420	.00411	.00382	.00346			

After Ahlvin and Ulery (1962).

Thí dụ

The flexible area shown in Figure 9.19 is uniformly loaded. Given that q = 150 kN/m^2, determine the vertical stress increase at point A.

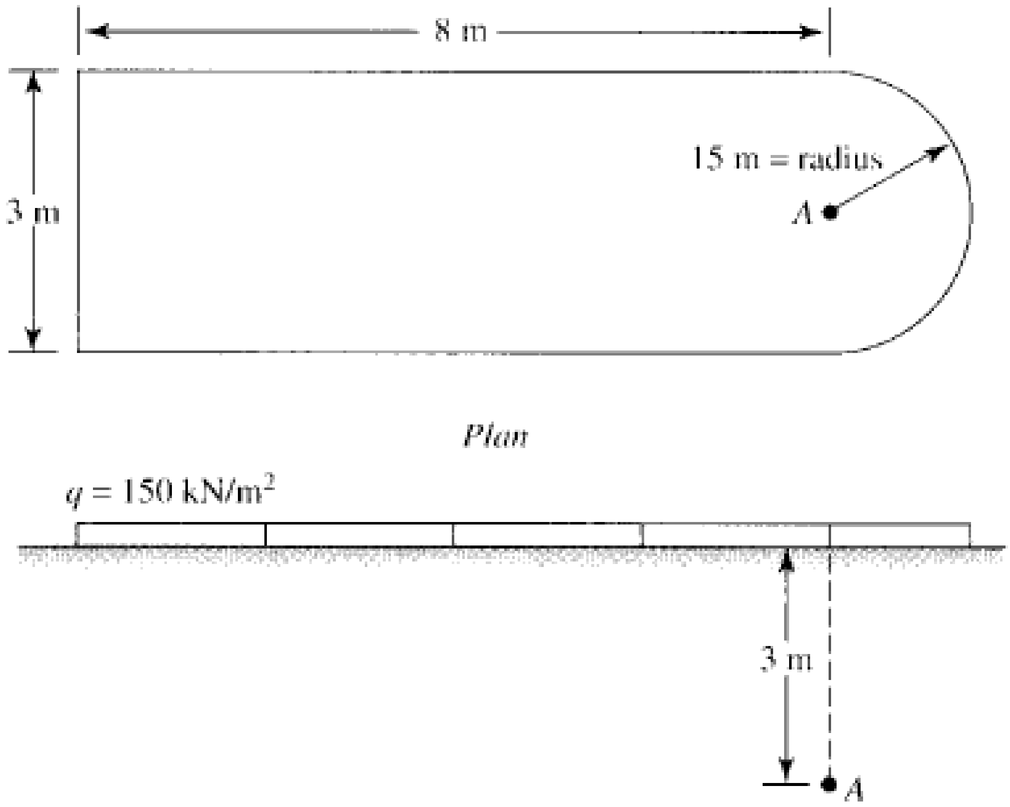
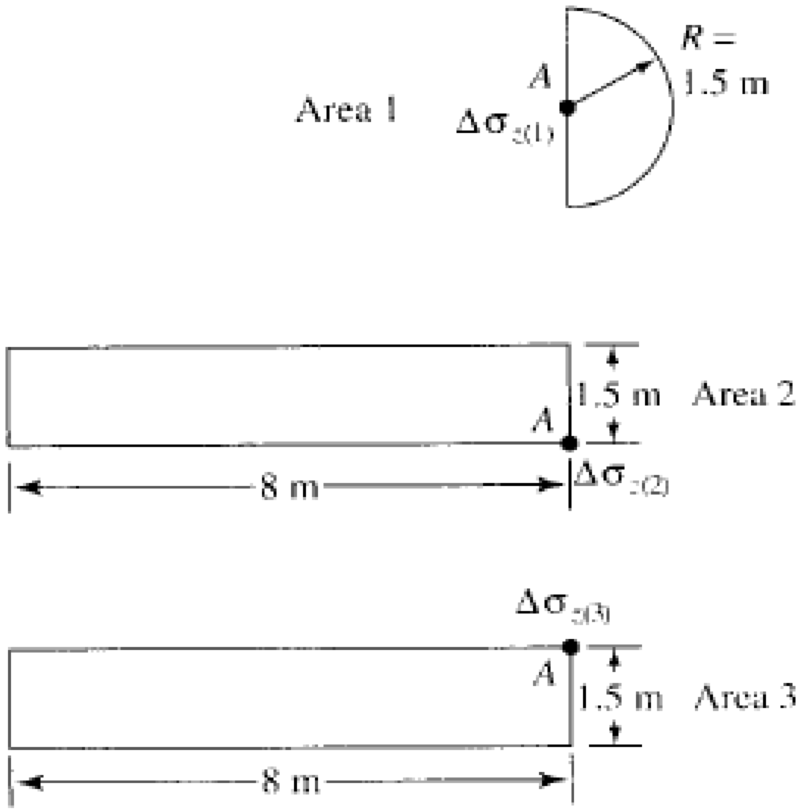


Figure 9.19 Uniformly loaded flexible area

Solution

$$\Delta\sigma_z = \Delta\sigma_{z(1)} + \Delta\sigma_{z(2)} + \Delta\sigma_{z(3)}$$



$$\Delta\sigma_{z(1)} = \left(\frac{1}{2}\right) q\left\{1 - \frac{1}{[(R/z)^2 + 1]^{3/2}}\right\}$$

$R = 1.5$ m, $z = 3$ m, and $q = 150$ kN/m^2

$$\Delta\sigma_{z(1)} = \frac{150}{2}\left\{1 - \frac{1}{[(1.5/3)^2 + 1]^{3/2}}\right\} = 21.3 \text{ kN/m}^2$$

$$\Delta\sigma_{z(2)} = \Delta\sigma_{z(3)}$$

$$m = \frac{1.5}{3} = 0.5 \qquad n = \frac{8}{3} = 2.67$$

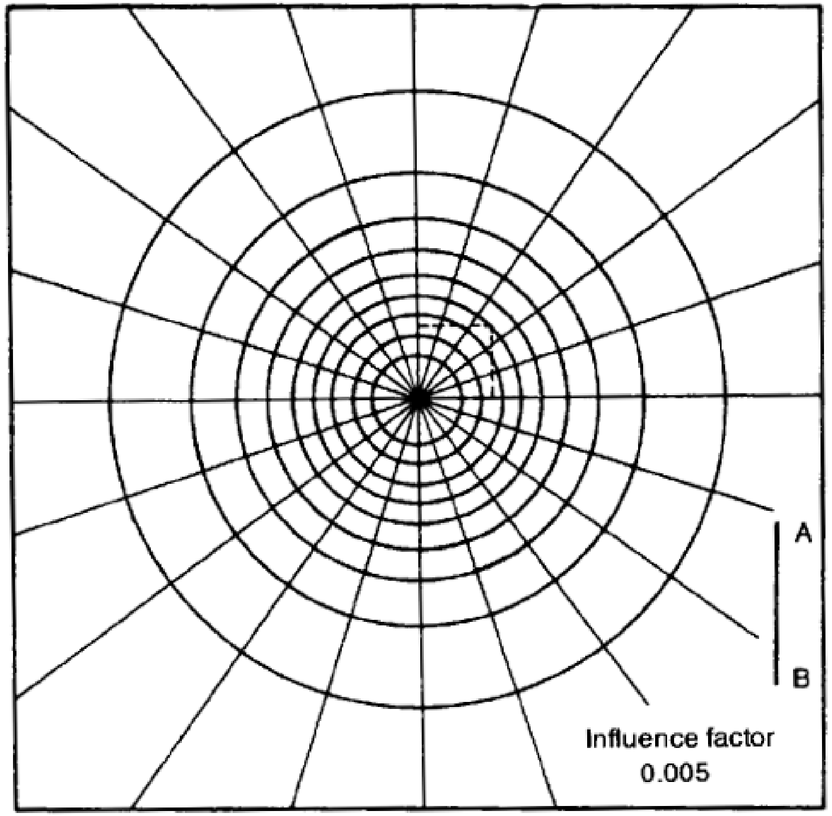
→ $I_3 = 0.1365.$

$$\Delta\sigma_{z(2)} = \Delta\sigma_{z(3)} = qI_3 = (150)(0.1365) = 20.48 \text{ kN/m}^2$$

$$\Delta\sigma_z = 21.3 + 20.48 + 20.48 = \mathbf{62.26\ kN/m^2}$$

2 Ứng suất

Ø Phương pháp biểu đồ Newmark



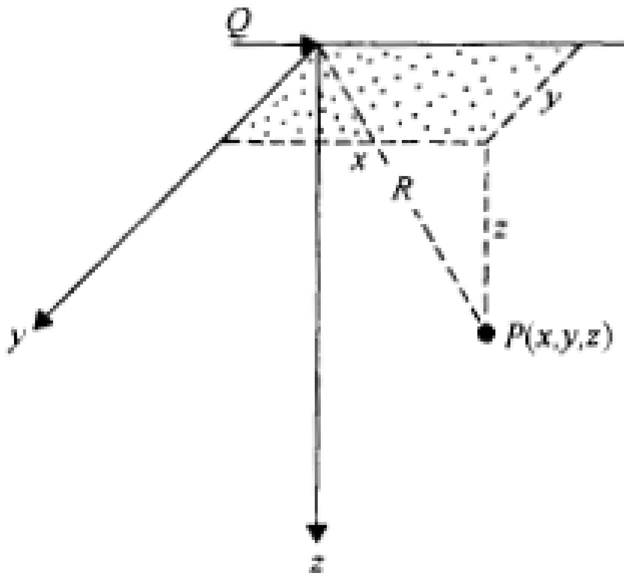
$$\Delta s_z = N \times q \times I_N$$

$$\sigma_z = p\left\{1 - \frac{1}{\left[1 + \left(\frac{r}{z}\right)^2\right]^{\frac{3}{2}}}\right\}$$

$$\frac{r}{z} = \sqrt{\left(1 - \frac{\sigma_z}{p}\right)^{-\frac{2}{3}} - 1}$$

$\frac{\sigma_z}{p}$	$\frac{r}{z}$
0.0	0.00
0.1	0.27
0.2	0.40
0.3	0.52
0.4	0.64
0.5	0.77
0.6	0.92
0.7	1.11
0.8	1.39
0.9	1.91
1.0	∞

2.3.10 Ứng suất do lực tập trung ngang



$$\sigma_z = \frac{3Qxz^2}{2\pi R^5}$$

$$\sigma_x = \frac{Q}{2\pi}\frac{x}{R^3}\left\{\frac{3x^2}{R^2} - (1-2\nu) + \frac{(1-2\nu)R^2}{(R+z)^2}\left[3 - \frac{x^2(3R+z)}{R^2(R+z)}\right]\right\}$$

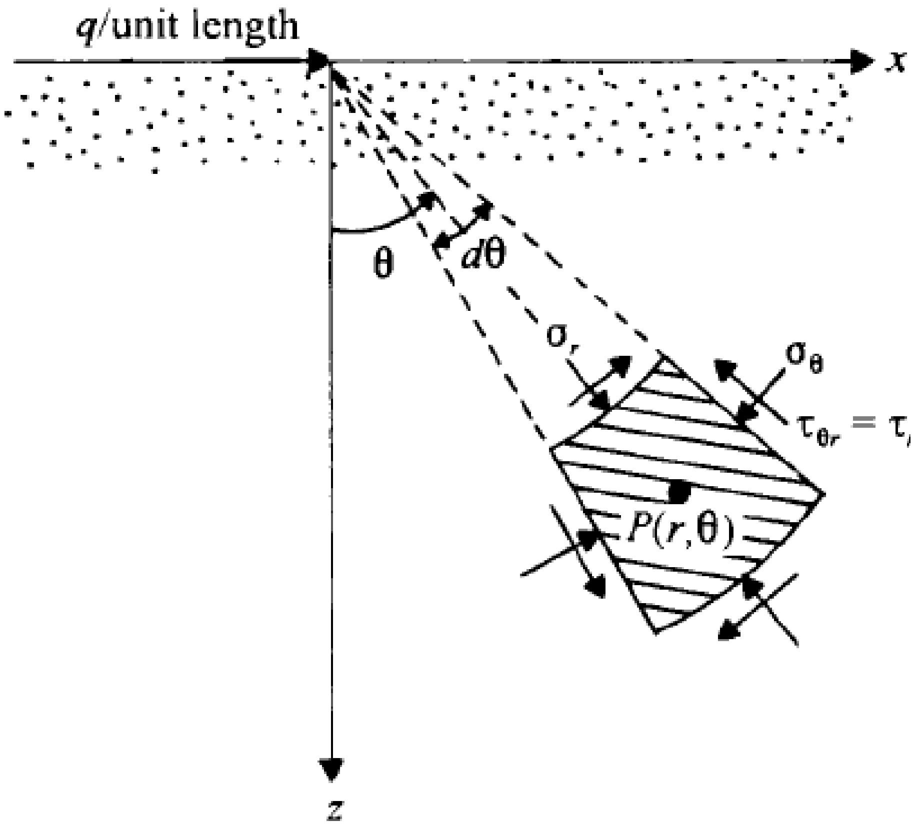
$$\sigma_y = \frac{Q}{2\pi}\frac{x}{R^3}\left\{\frac{3y^2}{R^2} - (1-2\nu) + \frac{(1-2\nu)R^2}{(R+z)^2}\left[3 - \frac{y^2(3R+z)}{R^2(R+z)}\right]\right\}$$

$$\tau_{xy} = \frac{Q}{2\pi}\frac{y}{R^3}\left\{\frac{3x^2}{R^2} + \frac{(1-2\nu)R^2}{(R+z)^2}\left[1 - \frac{x^2(3R+z)}{R^2(R+z)}\right]\right\}$$

$$\tau_{xz} = \frac{3Q}{2\pi}\frac{x^2z}{R^5}$$

$$\tau_{yz} = \frac{3Q}{2\pi}\frac{xyz}{R^5}$$

2.3.11 Ứng suất do tải trọng đường thẳng nằm ngang



$$\sigma_r = \frac{2q}{\pi r}\sin\theta$$

$$\sigma_\theta = 0$$

$$\tau_{r\theta} = 0$$

In the rectangular coordinate system,

$$\sigma_z = \frac{2q}{\pi}\frac{xz^2}{(x^2+z^2)^2}$$

$$\sigma_x = \frac{2q}{\pi}\frac{x^3}{(x^2+z^2)^2}$$

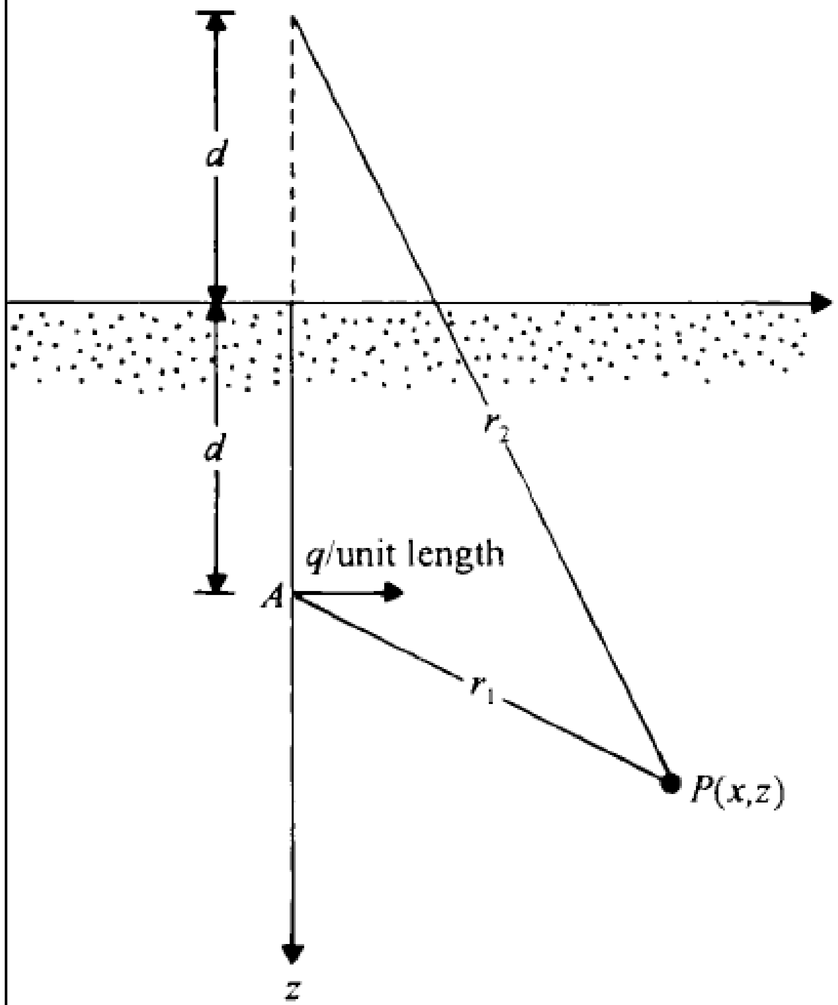
$$\sigma_y = \nu(\sigma_x + \sigma_z).$$

$$\tau_{xz} = \frac{2q}{\pi}\frac{x^2 z}{(x^2+z^2)^2}$$

Values of $\sigma_z/(q/z)$, $\sigma_x/(q/z)$, and $\tau_{xz}/(q/z)$

x/z	$\sigma_z/(q/z)$	$\sigma_x/(q/z)$	$\tau_{xz}/(q/z)$
0	0	0	0
0.1	0.062	0.0006	0.006
0.2	0.118	0.0049	0.024
0.3	0.161	0.0145	0.048
0.4	0.189	0.0303	0.076
0.5	0.204	0.0509	0.102
0.6	0.207	0.0743	0.124
0.7	0.201	0.0984	0.141
0.8	0.189	0.1212	0.151
0.9	0.175	0.1417	0.157
1.0	0.159	0.1591	0.159
1.5	0.090	0.2034	0.136
2.0	0.051	0.2037	0.102
3.0	0.019	0.1719	0.057

2.3.12 Ứng suất do tải trọng đường thẳng nằm ngang bên trong khối đất

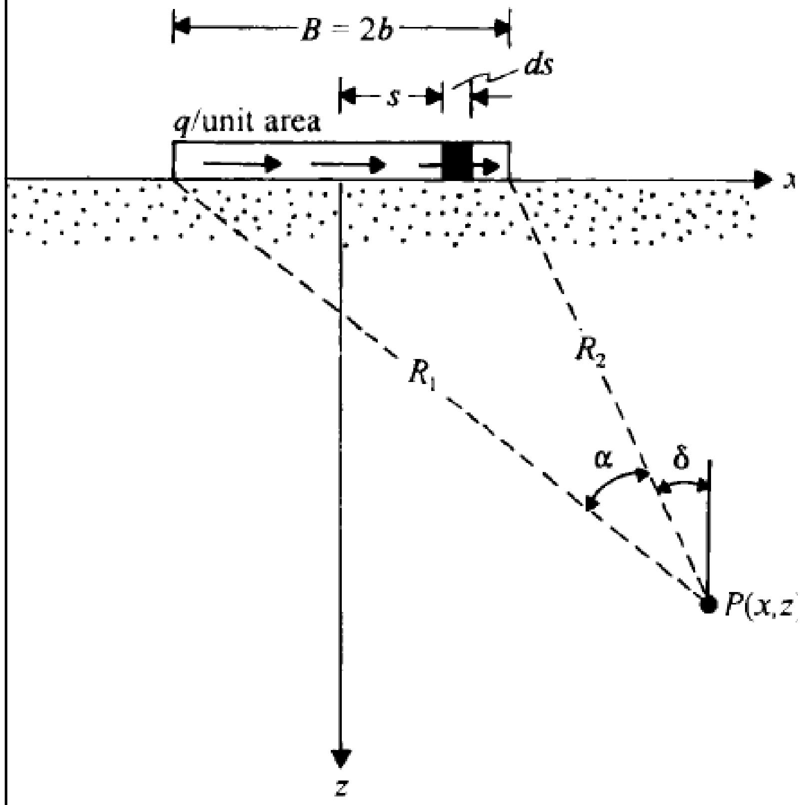


$$\sigma_z = \frac{qx}{\pi}\left\{\frac{1}{2(1-\nu)}\left[\frac{(z-d)^2}{r_1^4} - \frac{d^2 - z^2 + 6dz}{r_2^4} + \frac{8dz\,x^2}{r_2^6}\right] - \frac{1-2\nu}{4(1-\nu)}\left[\frac{1}{r_1^2} - \frac{1}{r_2^2} - \frac{4z(d+z)}{r_2^4}\right]\right\}$$

$$\sigma_x = \frac{qx}{\pi}\left\{\frac{1}{2(1-\nu)}\left[\frac{x^2}{r_1^4} + \frac{x^2 + 8dz + 6d^2}{r_2^4} + \frac{8dz(d+z)^2}{r_2^6}\right] + \frac{1-2\nu}{4(1-\nu)}\left[\frac{1}{r_1^2} + \frac{3}{r_2^2} - \frac{4z(d+z)}{r_2^4}\right]\right\}$$

$$\tau_{xz} = \frac{q}{\pi}\left\{\frac{1}{2(1-\nu)}\left[\frac{(z-d)x^2}{r_1^4} + \frac{(2dz + x^2)(d+z)}{r_2^4} - \frac{8dz(d+z)x^2}{r_2^6}\right] + \frac{1-2\nu}{4(1-\nu)}\left[\frac{z-d}{r_1^2} + \frac{3z+d}{r_2^2} - \frac{4z(d+z)^2}{r_2^4}\right]\right\}$$

2.3.13 Ứng suất do tải trọng phân bố đều hình bằng nằm ngang



$$\sigma_z = \int d\sigma_z = \frac{2q}{\pi}\int_{s=-b}^{s=+b} \frac{(x-s)z^2}{[(x-s)^2+z^2]^2}\,ds$$

$$= \frac{4bqxz^2}{\pi[(x^2+z^2-b^2)^2+4b^2z^2]}$$

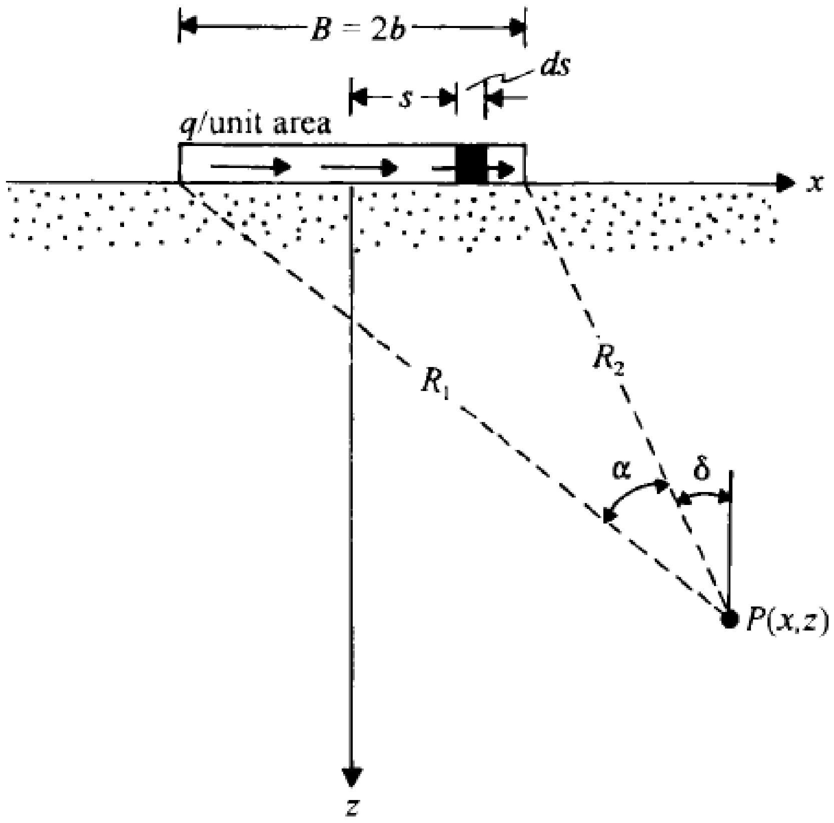
$$\sigma_x = \int d\sigma_x = \frac{2q}{\pi}\int_{s=-b}^{s=+b} \frac{(x-s)^3}{[(x-s)^2+z^2]^2}\,ds$$

$$= \frac{q}{\pi}\left[2.303\log\frac{(x+b)^2+z^2}{(x-b)^2+z^2} - \frac{4bxz^2}{(x^2+z^2-b^2)^2+4b^2z^2}\right]$$

$$\tau_{xz} = \int d\tau_{xz} = \frac{2q}{\pi}\int_{s=-b}^{s=+b} \frac{(x-s)^2z}{[(x-s)^2+z^2]^2}\,ds$$

$$= \frac{q}{\pi}\left[\tan^{-1}\frac{z}{x-b} - \tan^{-1}\frac{z}{x+b} + \frac{2bz(x^2-z^2-b^2)}{(x^2+z^2-b^2)^2+4b^2z^2}\right]$$

Ứng suất

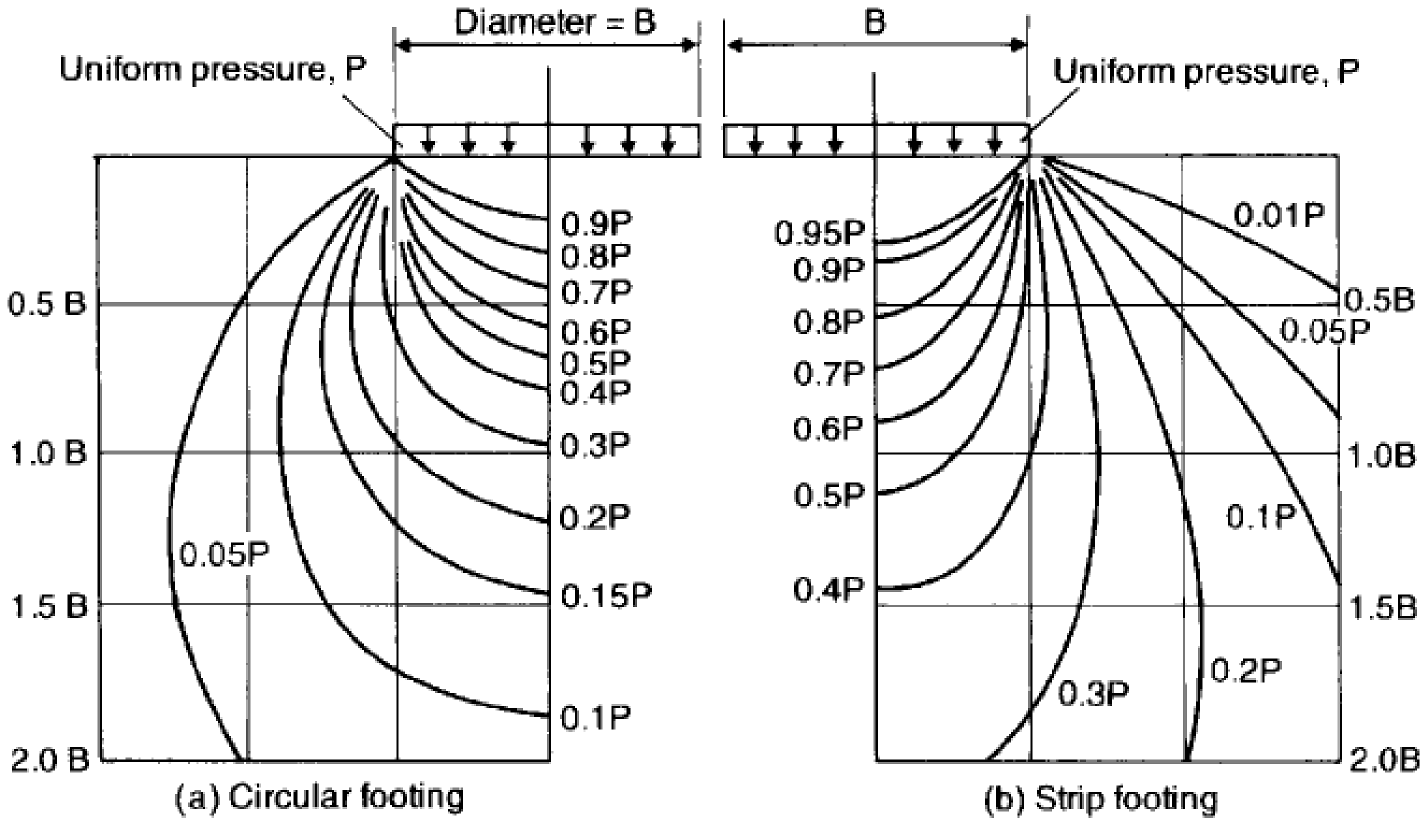


$$\sigma_z = \frac{q}{\pi}[\sin\alpha \sin(\alpha + 2\delta)]$$

$$\sigma_x = \frac{q}{\pi}\left[2.303 \log \frac{R_1^2}{R_2^2} - \sin\alpha \sin(\alpha + 2\delta)\right]$$

$$\tau_{xz} = \frac{q}{\pi}[\alpha - \sin\alpha \cos(\alpha + 2\delta)]$$

2.3.14 Quả Bầu Đồng Ứng suất



2 Ứng suất

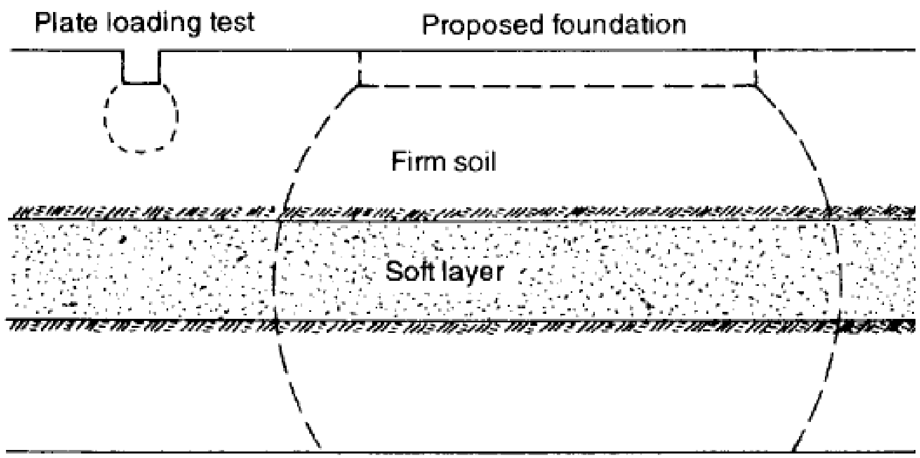
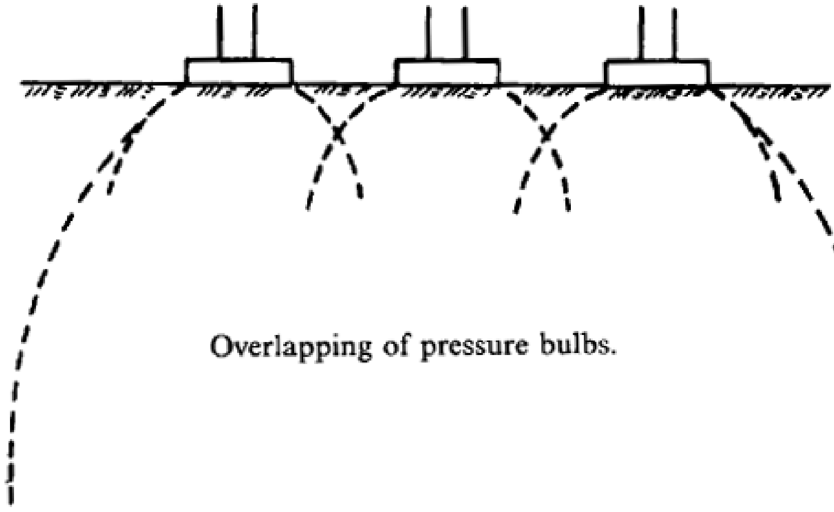
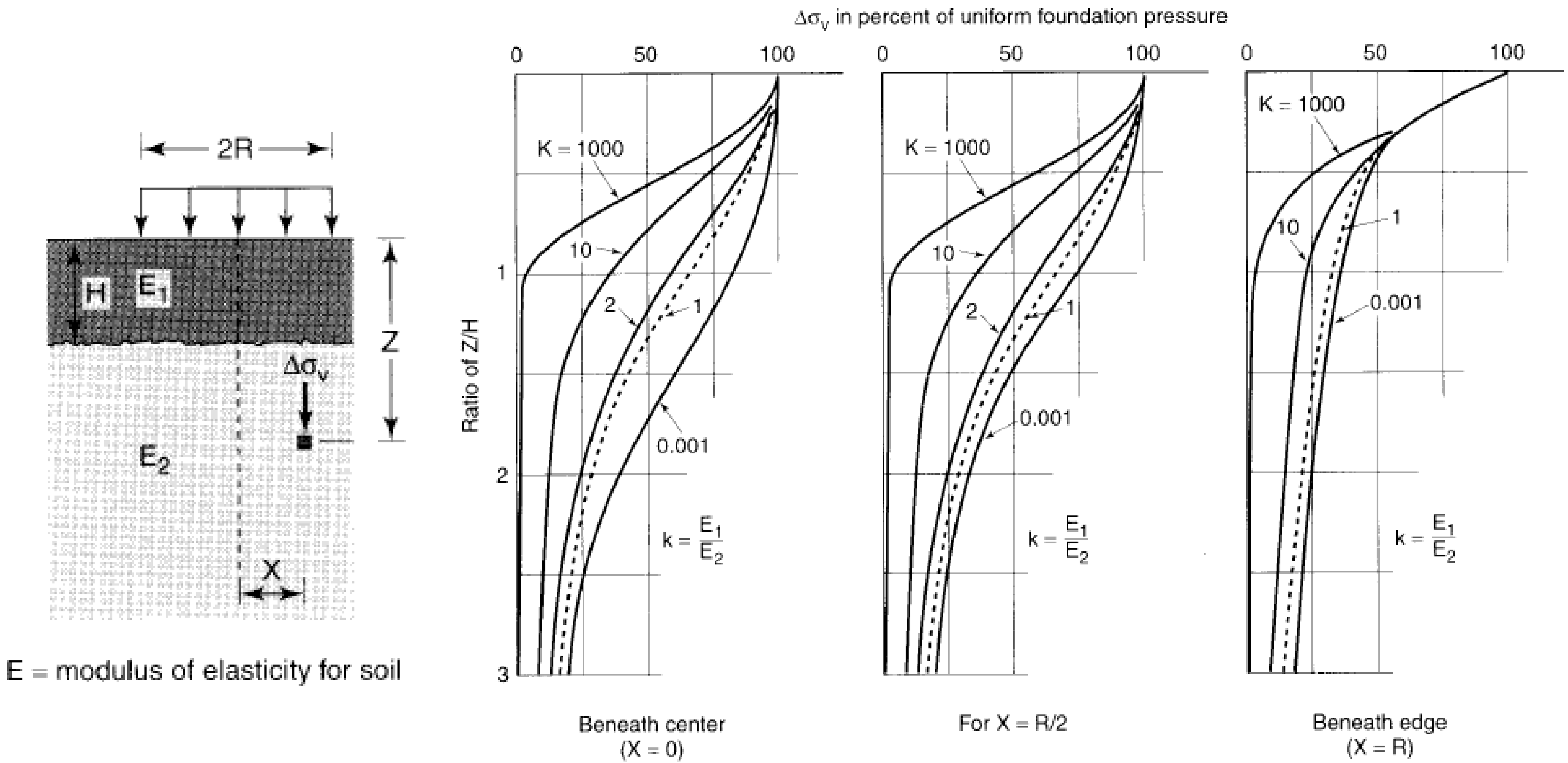


Illustration of how a plate loading test may give misleading results.



Overlapping of pressure bulbs.

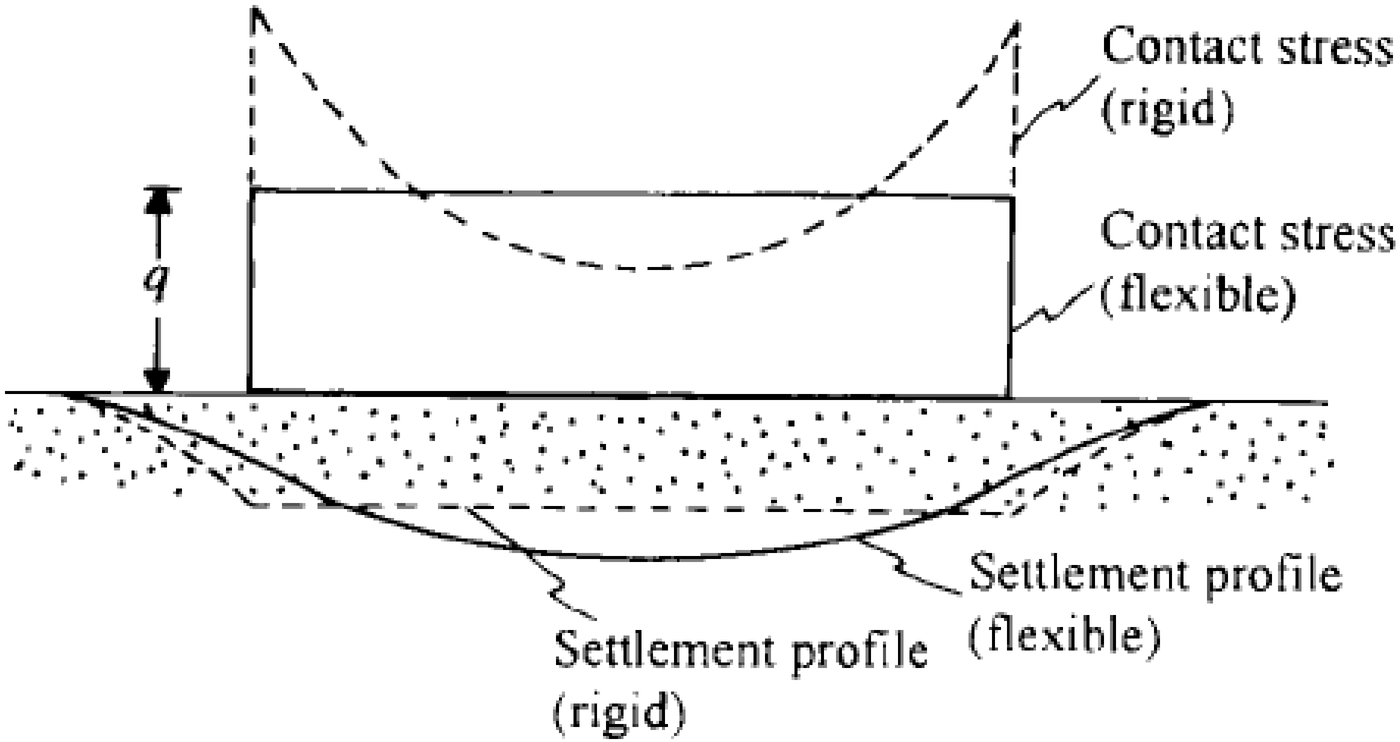
2.4 Ứng suất trong nền không đồng nhất



Sự thay đổi ứng suất bên dưới móng tròn (H = R)

2.5 Ứng suất dưới đáy móng

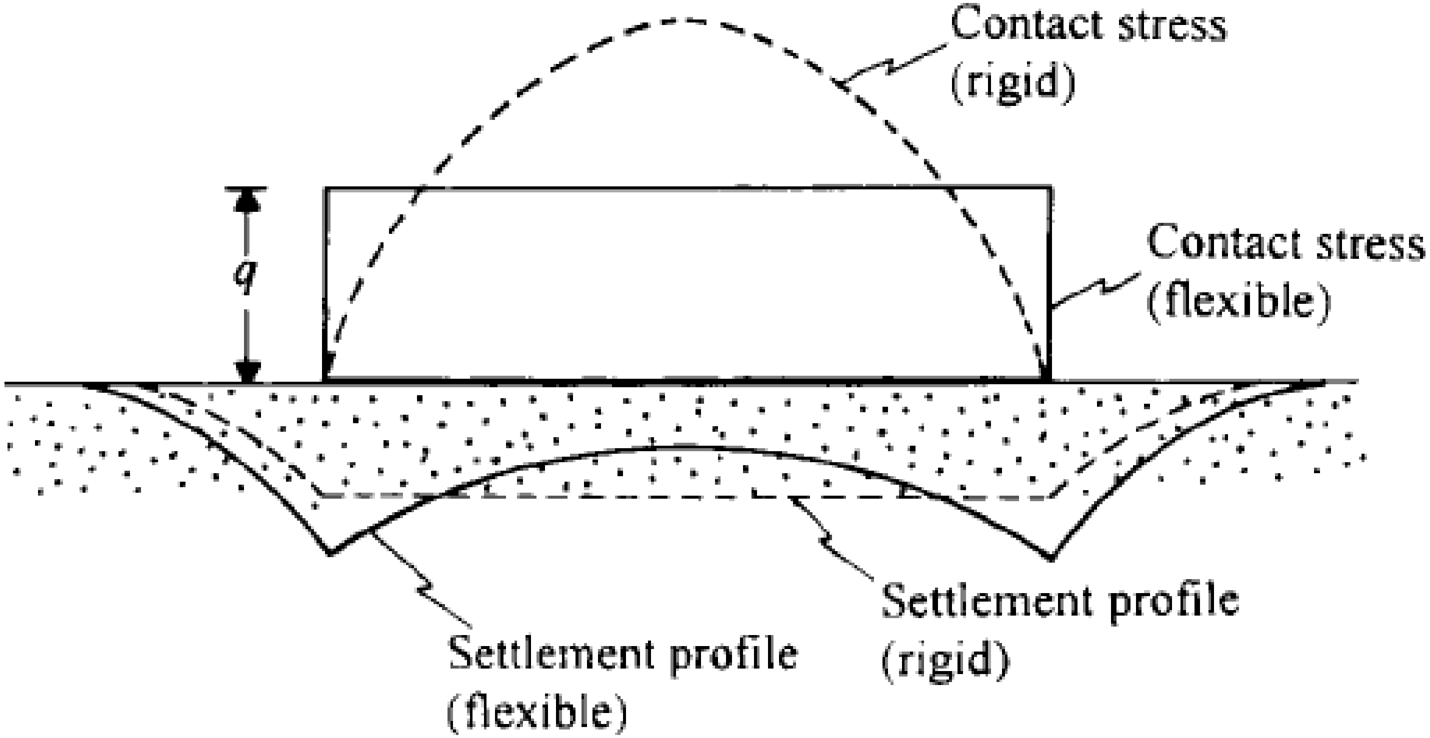
Ø Đất sét



Contact pressure and settlement profiles for foundations on clay.

2 Ứng suất

Ø Đất cát



Contact pressure and settlement profiles for foundations on sand.